Course compass

Question directrice : How does an integral turn a rate into a total, how do we read ∫ and dt, and why does area under a curve have physical meaning?

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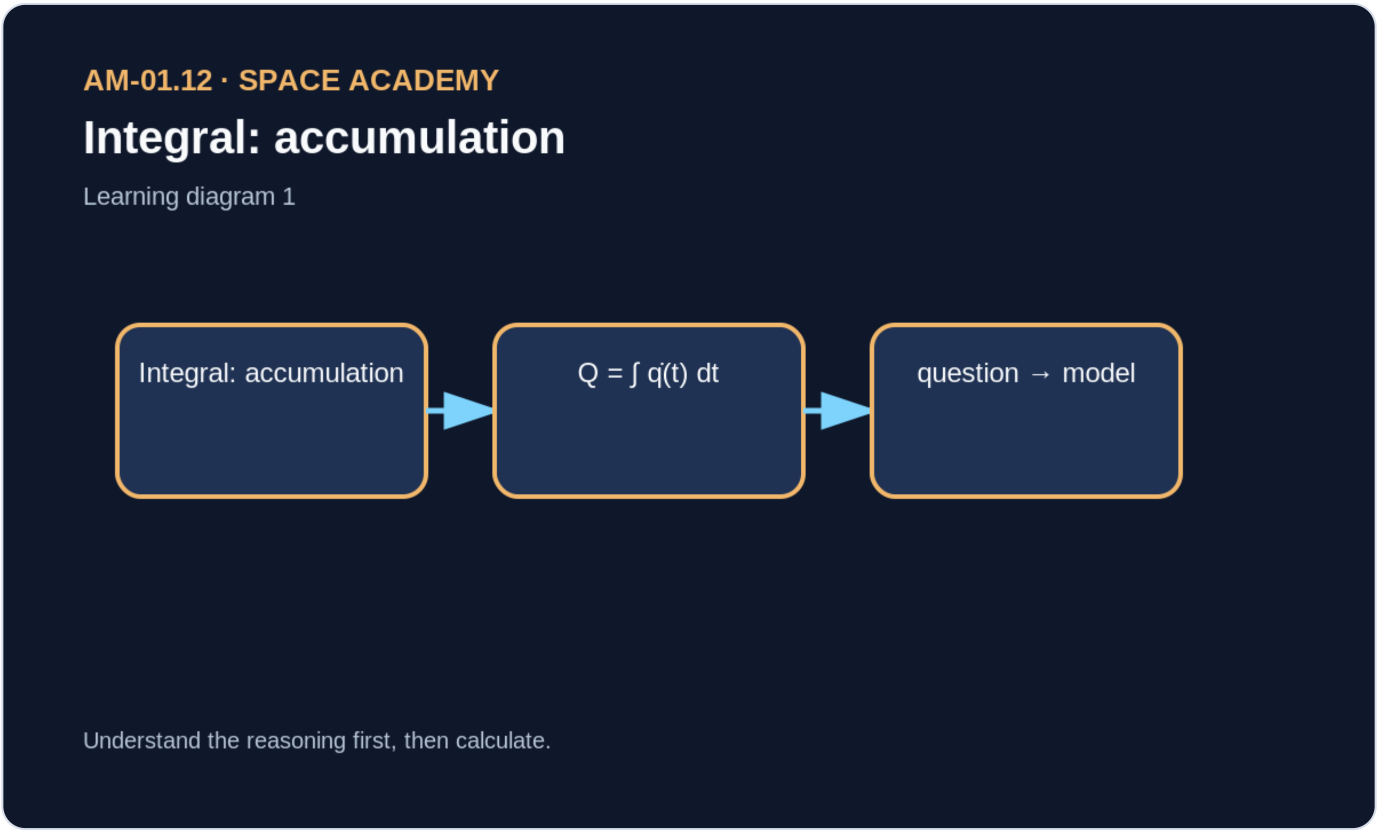
1 — An integral first answers a simple question: “how much in total?”

A unit produces water at a certain flow rate. Knowing the rate at each instant is not enough if the crew wants to know **how many kilograms were produced in total**.

An **integral** is an accumulation tool. It adds many small contributions.



A rate says how much per unit time; an integral reconstructs the total.

2 — How do we read ∫, dt and the limits?

∫ is read “integral”. $\int q(t)\,dt$ is read “the integral of q of t, d t”.

$\int_0^4 q(t)\,dt$ accumulates from t=0 to t=4. The small dt reminds us that the sum is made from very small pieces of time.

3 — Complete example 1: constant flow

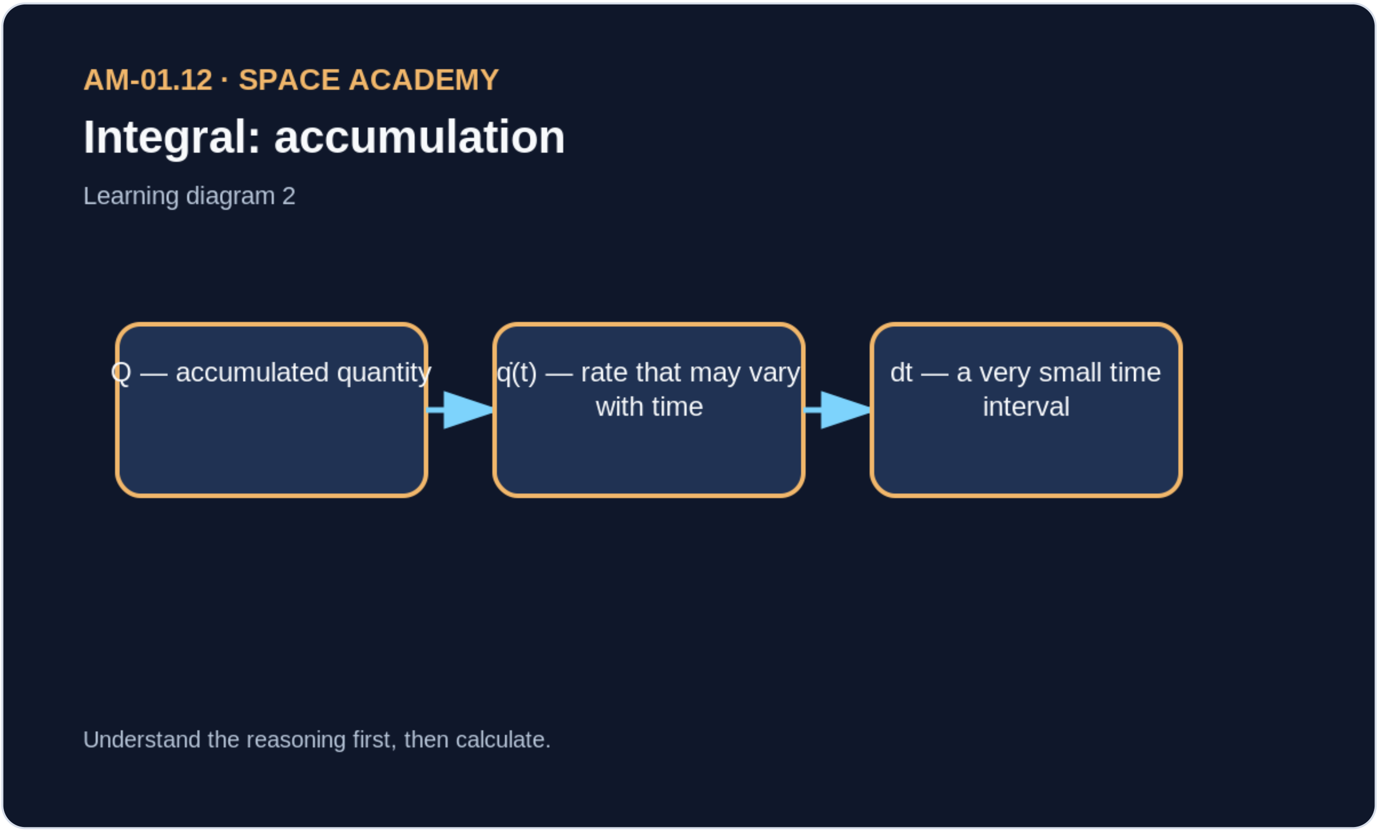
A unit produces 2 kg of water per hour for 5 h.

```
rate = 2 kg/h time = 5 h total = 2 × 5 = 10 kg
```

On a graph this is a rectangle: height 2 kg/h, width 5 h, area 10 kg.

Why do hours disappear?

(kg/h)×h=kg.



Area equals rate times duration equals total amount.

4 — Complete example 2: two flow levels

For 2 h the system produces 3 kg/h; for the next 3 h it produces 1 kg/h.

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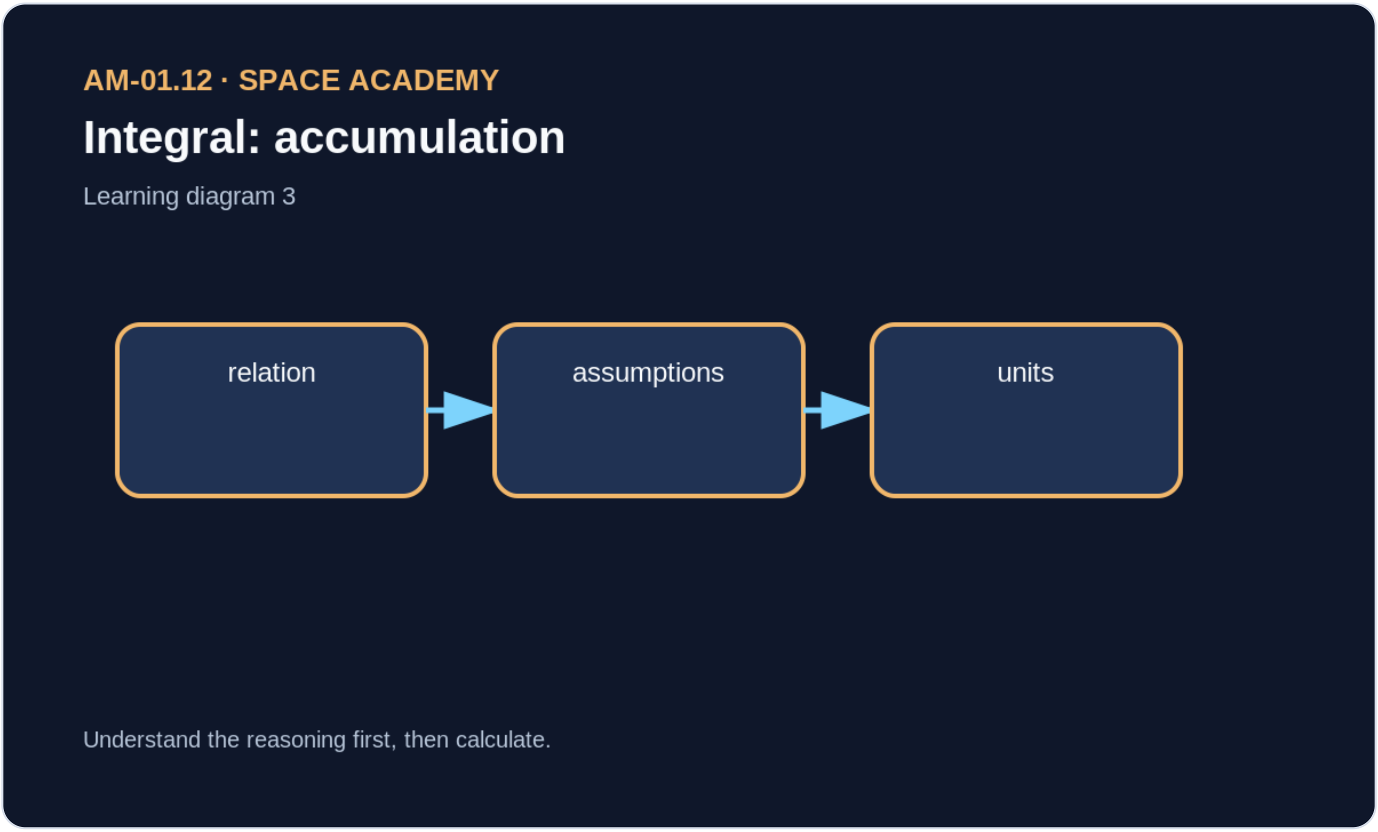
first period: 3×2=6 kg second period: 1×3=3 kg total=9 kg
```

The integral adds the two contributions.



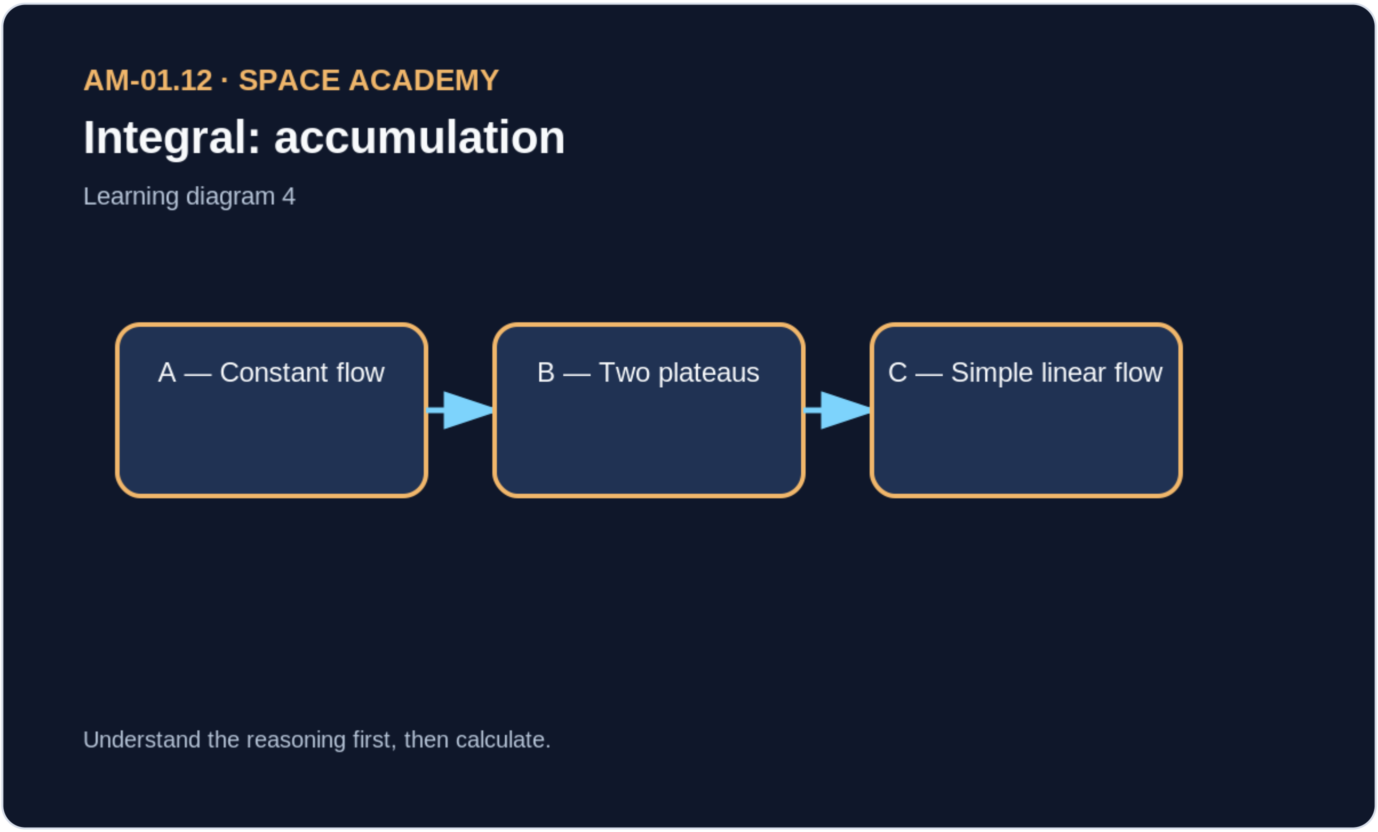
Split time into pieces and add the amount from each piece.

5 — Complete example 3: gradually increasing flow

Take the simple model `q(t)=t` kg/h from 0 to 4 h. The graph is a triangle with base 4 h and height 4 kg/h.

```
area = 4×4÷2 = 8 kg
```

$$\int_0^4 t\,dt = 8$$



The integral links a flow curve to total accumulated quantity.

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6 — Why “sum of small rectangles”?

For an irregular rate, split time into small intervals. On each interval:

$$\text{small amount} \approx \text{rate} \times \text{small duration}$$

Add all small amounts. The integral formalizes the limit of this idea.

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7 — Integral and derivative answer opposite questions

The derivative asks: **how fast is the total changing now?** The integral asks: **if I know the rate, how much has accumulated?**

If the derivative of stored water is water flow, integrating the flow over time recovers the change in stored water.

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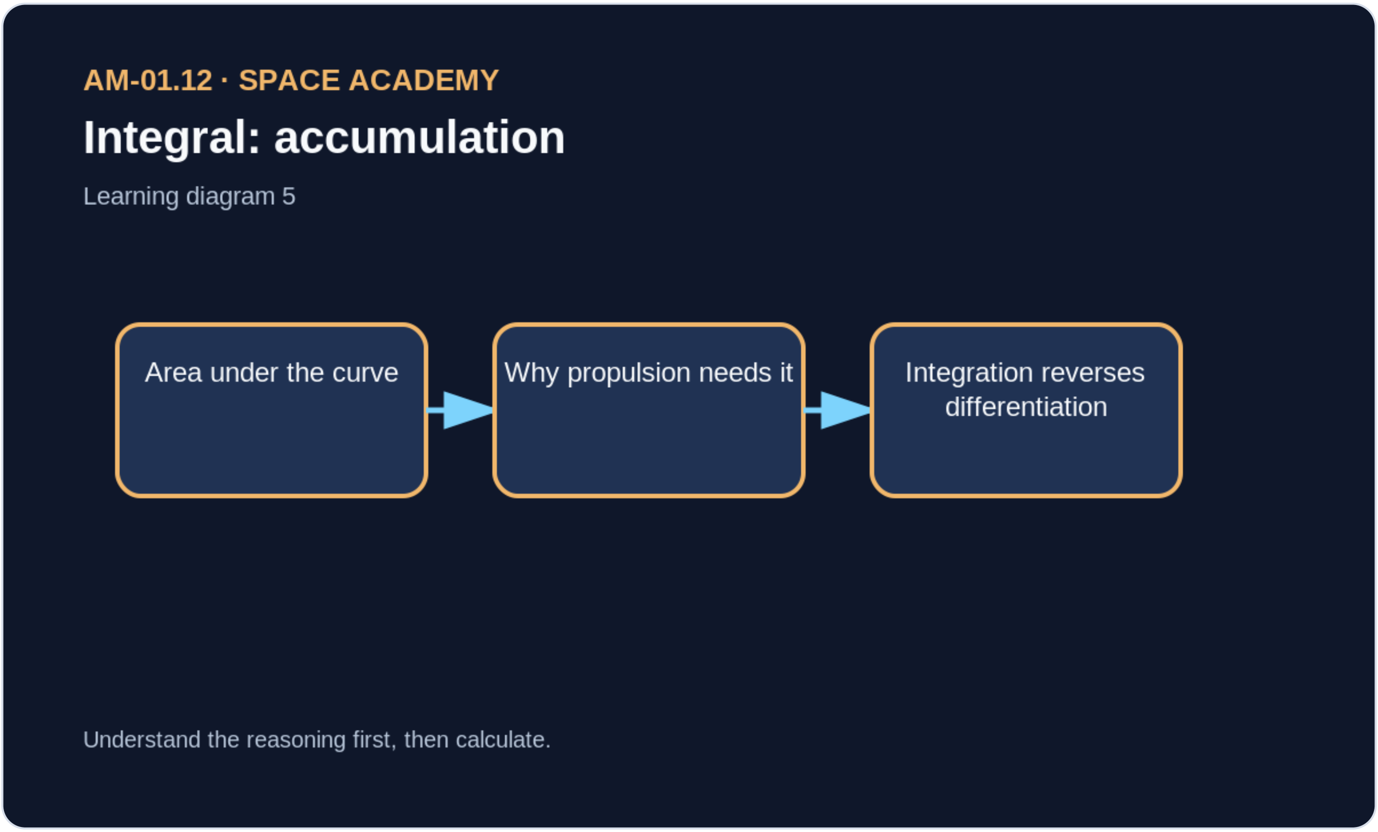
8 — Real data often mean a numerical sum

A computer may receive one flow measurement every second rather than a perfect function. It then uses rectangles, trapezoids or other numerical methods.

Missing samples, sensor noise and timing errors can accumulate into a significant error in the total.



Numerical integration adds contributions from successive measurements.

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9 — Three cases to recognize

1. **Constant rate:** rate × time.
2. **Steps:** compute each rectangle and add.
3. **Simple curve:** use a known area formula or an integral function once the model is defined.

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10 — What to remember

- An integral calculates **accumulation**.
- ∫ is read “integral”.
- dt indicates accumulation over tiny pieces of time.
- Area under a flow-rate curve represents a total amount.
- Units must work: (kg/h)×h=kg.
- Derivative and integral answer inverse questions.

Exercises and answers

Exercise 1

4 kg/h for 3 h: total?

12 kg.

Exercise 2

2 kg/h for 2 h then 5 kg/h for 1 h: total?

9 kg.

Exercise 3

Why does integrating kg/s over seconds give kg?

Because (kg/s)×s=kg.

Primary and technical sources

- NIST — SI Units
- NASA Johnson — mission engineering context