Course compass

Question directrice : What is a derivative, how do we read f prime of x and dx/dt, and how does average slope become instantaneous change?

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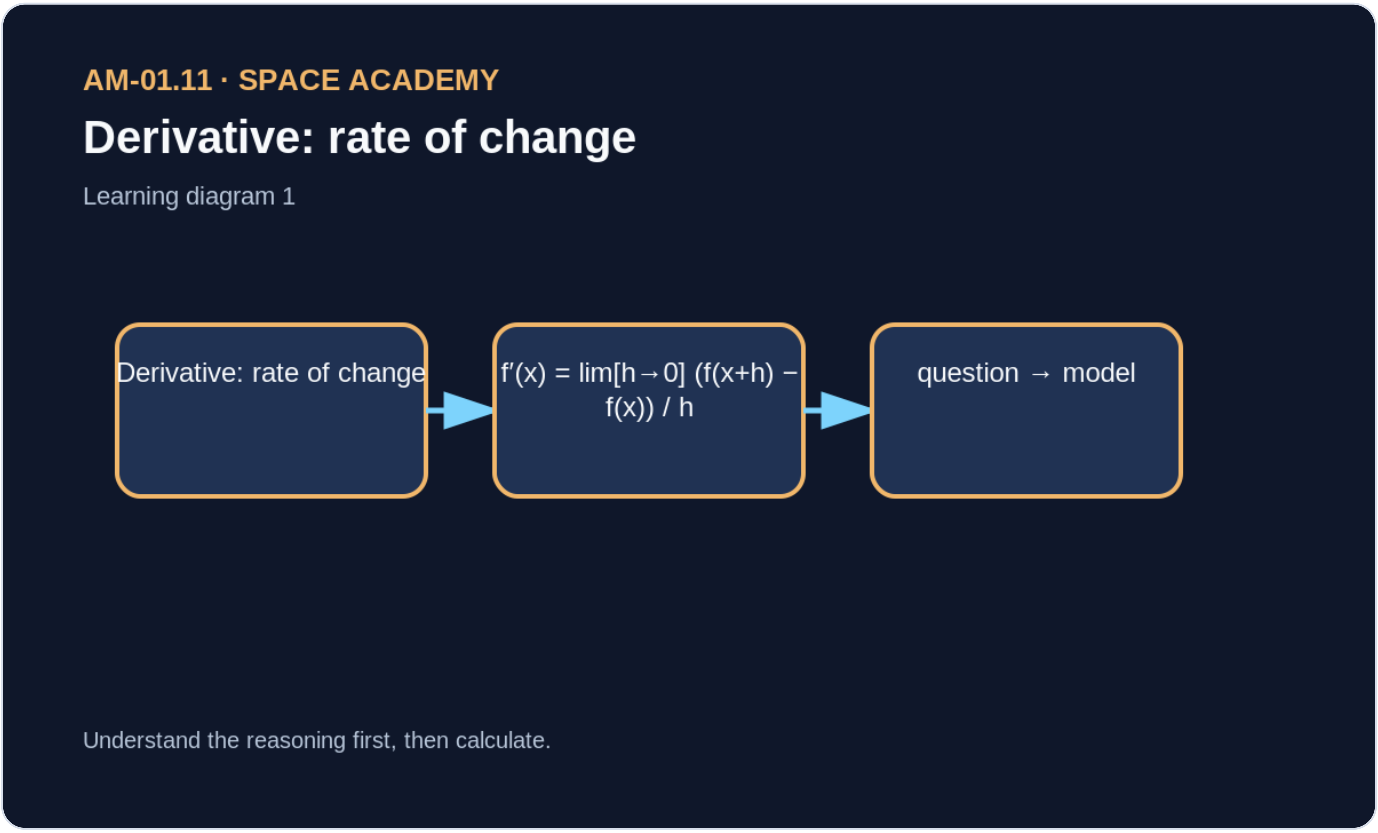
1 — The derivative starts with a speedometer question: “how fast is it changing now?”

A rover covers 100 m in 20 s, giving an average speed of 5 m/s. That does not say whether it was moving at 2 m/s first and 8 m/s later.

A **derivative** asks a more local question: **how fast is a quantity changing at this instant?**



A derivative looks for local change, not only the average over the whole trip.

2 — How do we read f′(x) and dx/dt?

`f′(x)` is read **“f prime of x”**. The prime mark indicates the derivative.

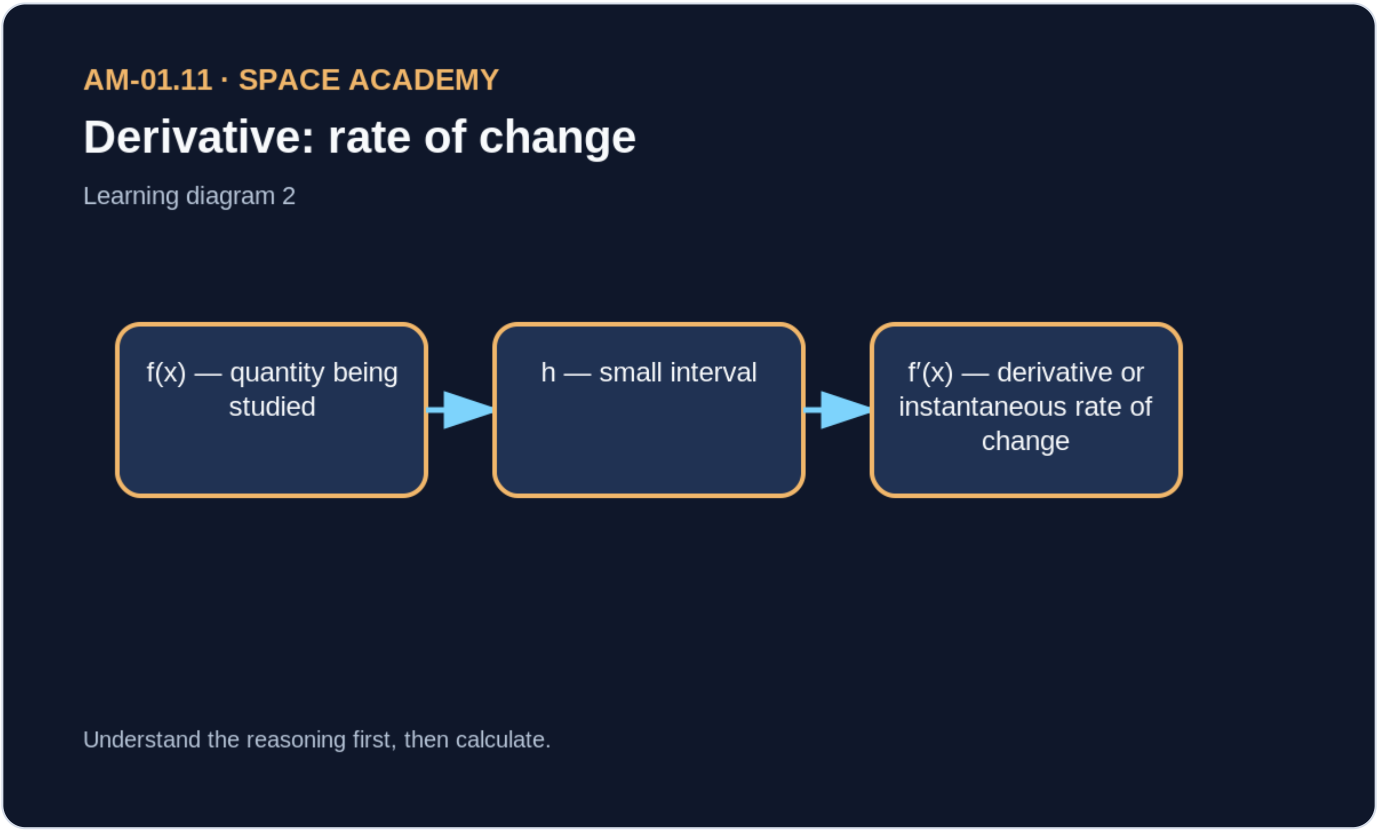
`dx/dt` is read “d x over d t” or “the derivative of x with respect to time t”. If x is metres and t seconds, dx/dt has units m/s.

3 — From average slope to local slope

$$\text{average velocity} = \Delta x \div \Delta t$$

To estimate velocity at one instant, choose measurements closer and closer around that instant. Average slope then approaches local slope.

> **A derivative is not division by zero.** We study what happens as the interval becomes very small; we do not simply put zero in the denominator.

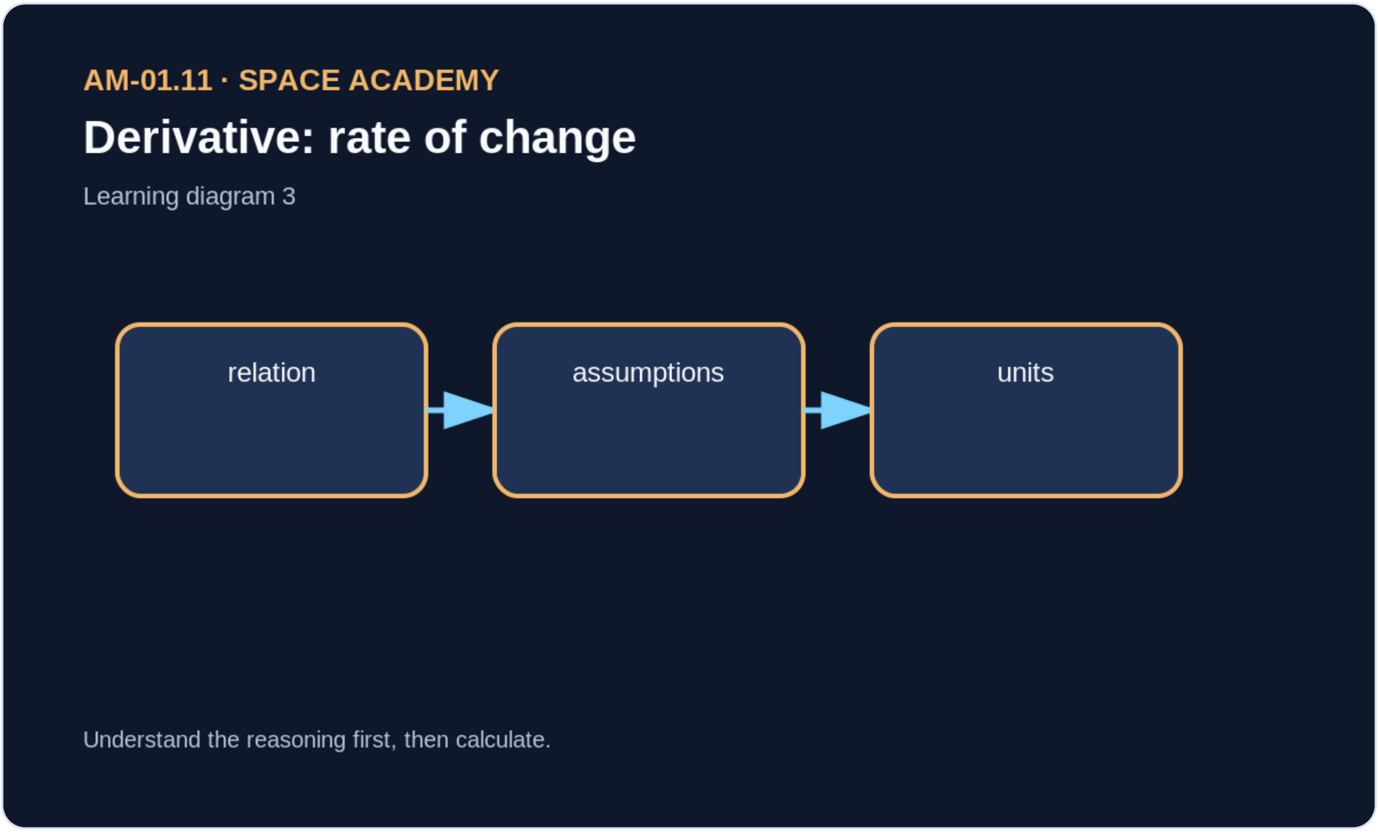


Closer points make the average slope approach the local slope.

4 — Complete example 1: $x(t)=2t^2$

$x(t)=2t^2$ is read “x of t equals two t squared”. Its derivative is $dx/dt=4t$.

At t=3 s: $v=4\times3=12$ m/s . Around the third second, position increases at 12 metres per second in this model.



The slope of position versus time gives instantaneous velocity.

5 — Complete example 2: falling tank pressure

Pressure falls from 300 kPa to 294 kPa between t=10 s and t=12 s.

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ΔP = 294 − 300 = −6 kPa Δt = 2 s average rate = −6 ÷ 2 = −3 kPa/s
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The negative sign means pressure is decreasing.

6 — Complete example 3: power ramp

Electrical power rises from 2 kW to 5 kW in 6 s.

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ΔP = 3 kW Δt = 6 s average rate = 0.5 kW/s
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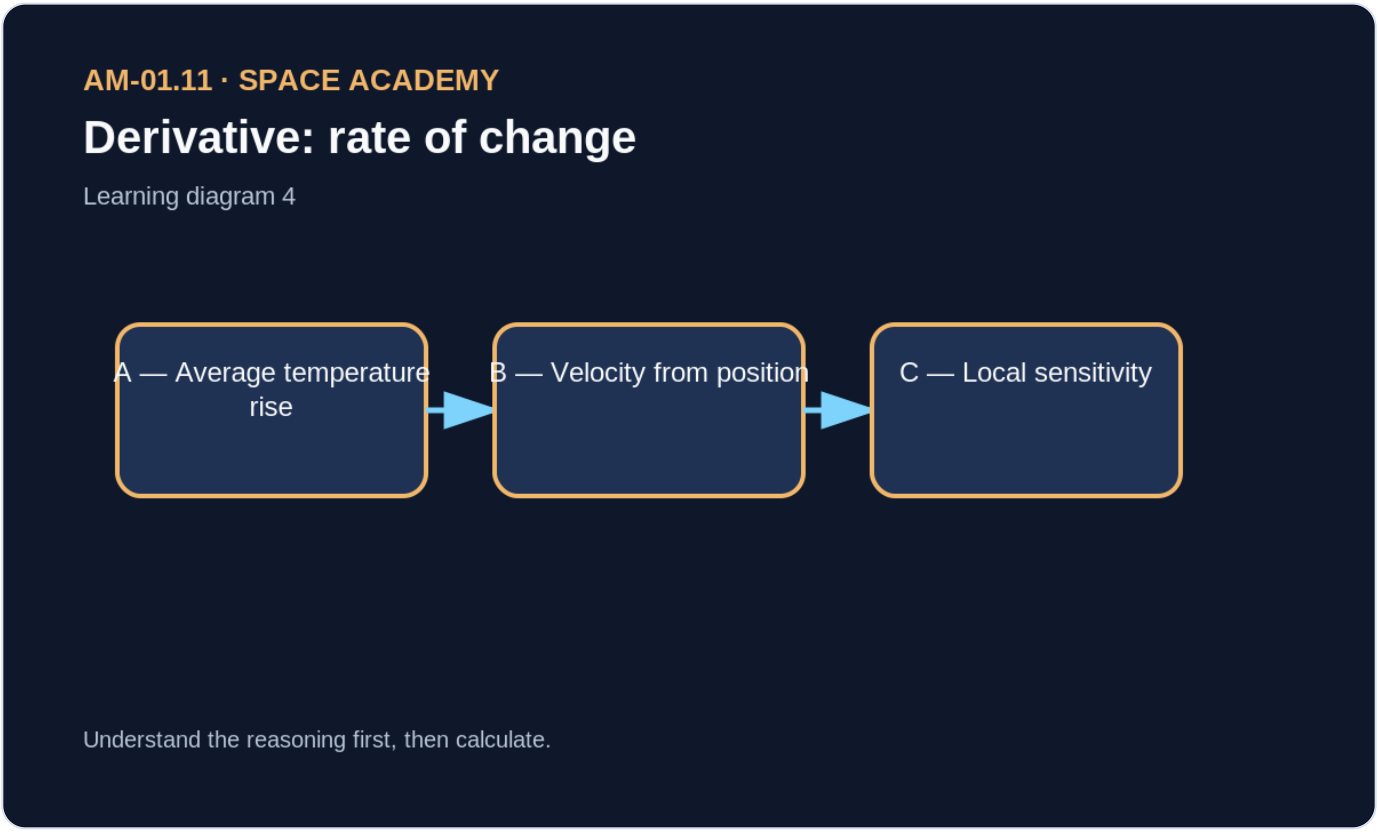
An instantaneous derivative would reveal whether the ramp is smooth or includes a spike.

7 — Units explain the derivative

Position m / time s → velocity m/s. Velocity m/s / time s → acceleration m/s². Pressure kPa / time s → kPa/s.



Units reveal what physical rate is being measured.

8 — Real measurements: derivatives can amplify noise

Small sensor fluctuations can produce large apparent differences when two very close samples are subtracted. Engineers therefore filter signals and choose time intervals carefully.

9 — Numerical estimate from two nearby measurements

If x=48.2 m at 9.9 s and x=50.6 m at 10.1 s:

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Δx=2.4 m Δt=0.2 s v≈2.4÷0.2=12 m/s
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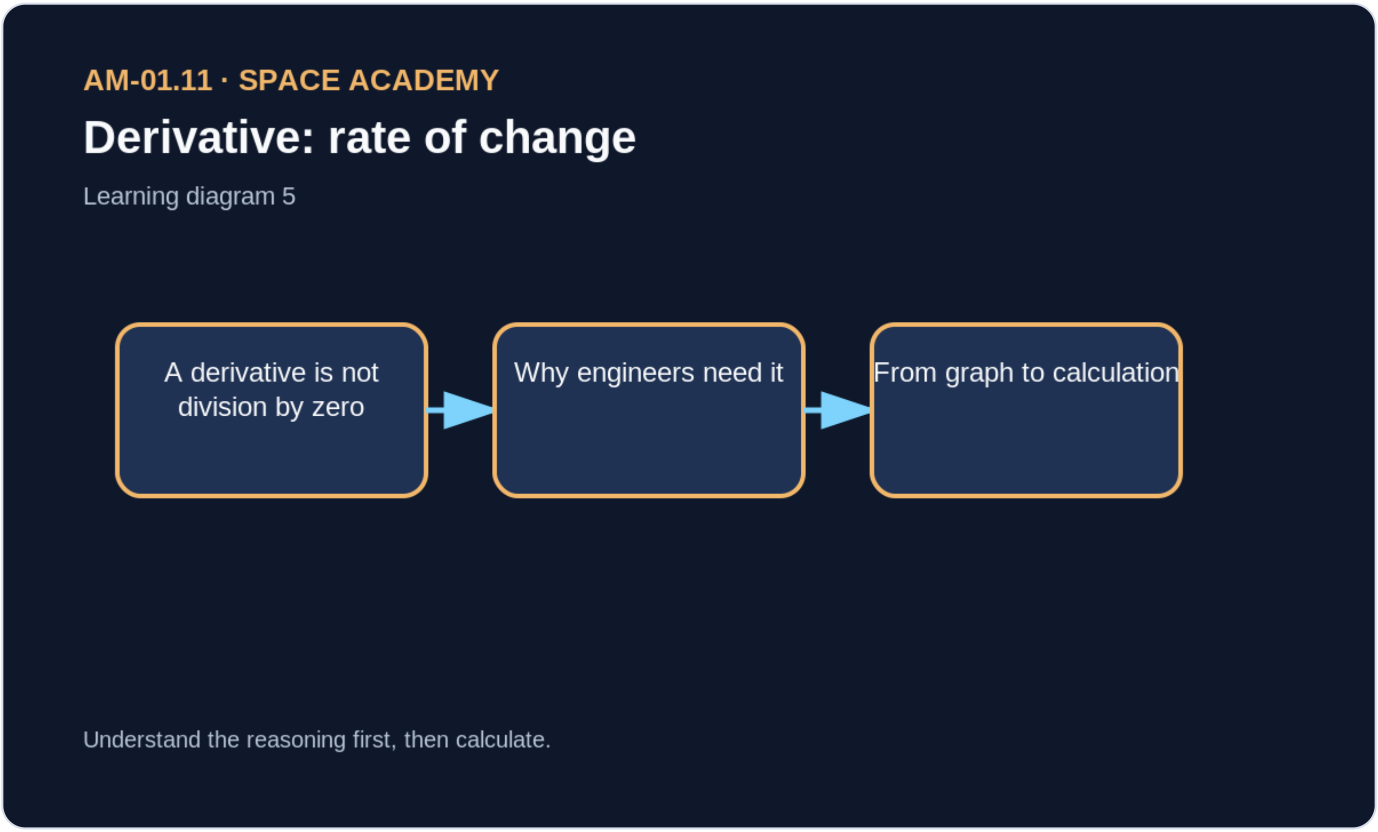
This estimates velocity around 10 s.



Nearby samples estimate a local rate if sensor noise is controlled.

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10 — What to remember

- A derivative measures a **local rate of change**.
- f′(x) is “f prime of x”.
- dx/dt is “d x over d t”.
- It extends average slope to a very small interval.
- Its units are quantity-units divided by variable-units.
- Real data require attention to noise and sampling.

Exercises and answers

Exercise 1

Position changes from 10 m to 16 m between 2 s and 4 s. Average velocity?

$(16-10)/(4-2)=3$ m/s.

Exercise 2

If $x(t)=3t^2$, derivative and value at $t=2$?

$dx/dt=6t$; at 2 s: 12 m/s if x is metres.

Exercise 3

Pressure falls 8 kPa in 4 s. Sign and rate?

Negative: -2 kPa/s.

Primary and technical sources

- NIST — SI Units
- NASA — navigation context