Course compass

Question directrice : What do e^x and ln do, how are they read, why are they inverse, and why does ln appear in the ideal rocket equation?

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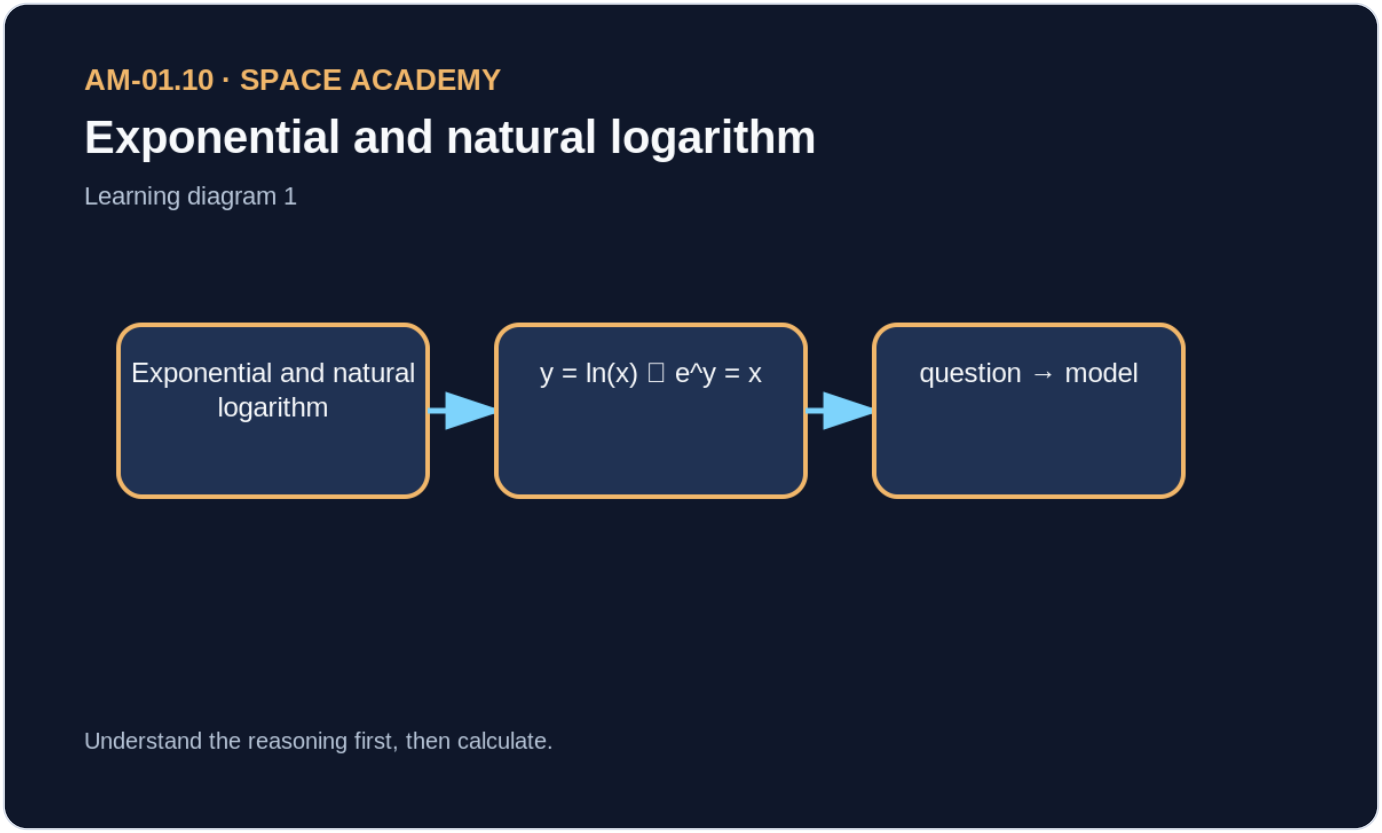
1 — Exponential change: multiply rather than add the same amount

If a reserve gains exactly 10 kg each hour, it grows by +10, +10, +10. **Exponential** growth is different: the change depends on what is already there. A culture that doubles goes 1, 2, 4, 8, 16.



Adding the same amount and multiplying by the same factor create different curves.

2 — What does e^x mean and how is it read?

e^x is read **“e to the power x”**. The number e is approximately 2.71828 and appears naturally in continuous-change models.

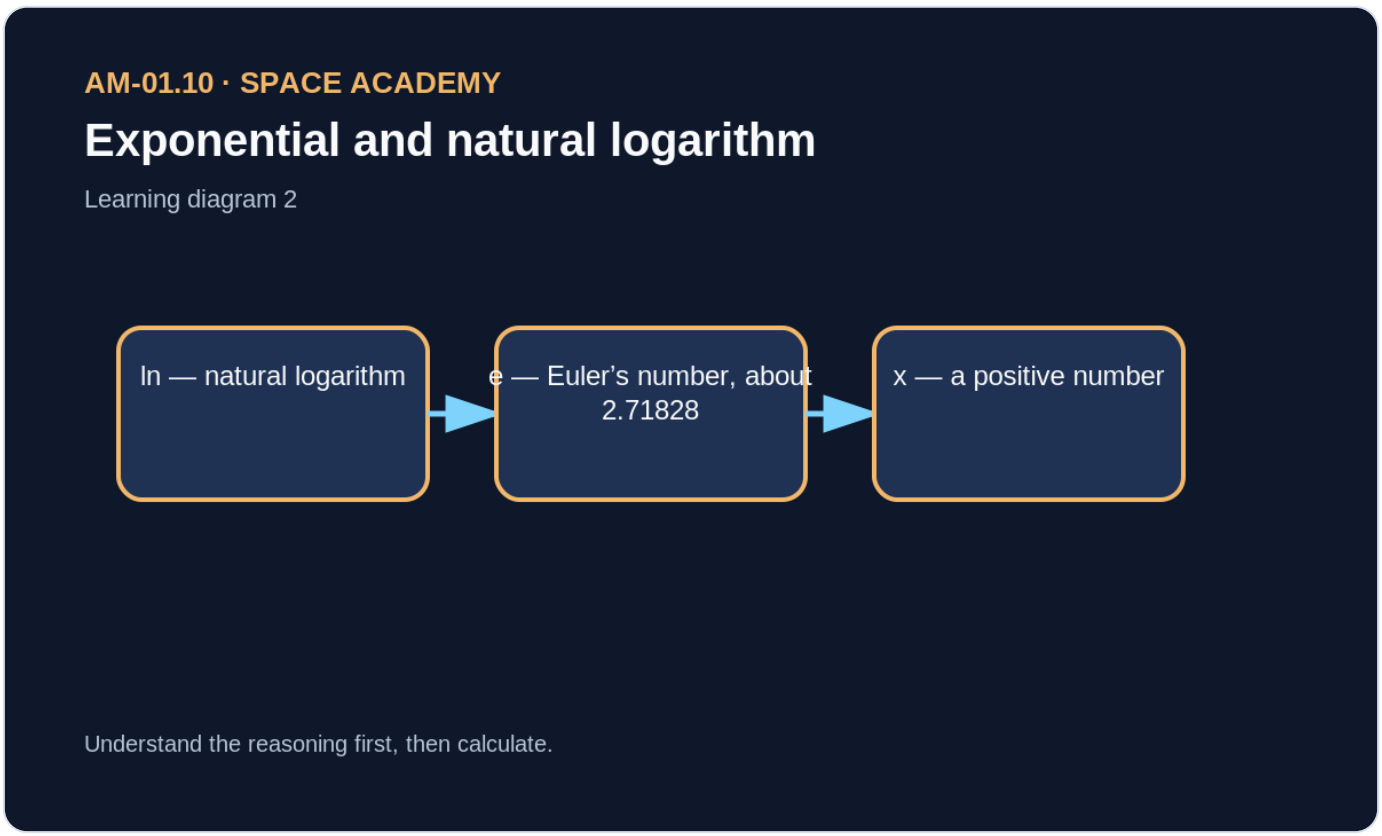
You do not need to memorize its digits. Think of e^x as turning an exponent x into a multiplication factor.

3 — Natural logarithm: the inverse question

If we know the final factor and want the exponent that produced it, the **natural logarithm**, written **ln**, answers that question.

$$\text{if } e^x = N, \text{ then } x = \ln(N)$$

Read: “if e to the power x equals N, then x equals the natural logarithm of N.”



The exponential goes from exponent to factor; the logarithm goes back from factor to exponent.

4 — Concrete example 1: how many doublings?

A culture goes 1 → 2 → 4 → 8. Reaching 8 takes three doublings because $2^3=8$.

Using logs: $x = \ln(8) \div \ln(2) \approx 2.079 \div 0.693 \approx 3$.

Why divide by ln(2)?

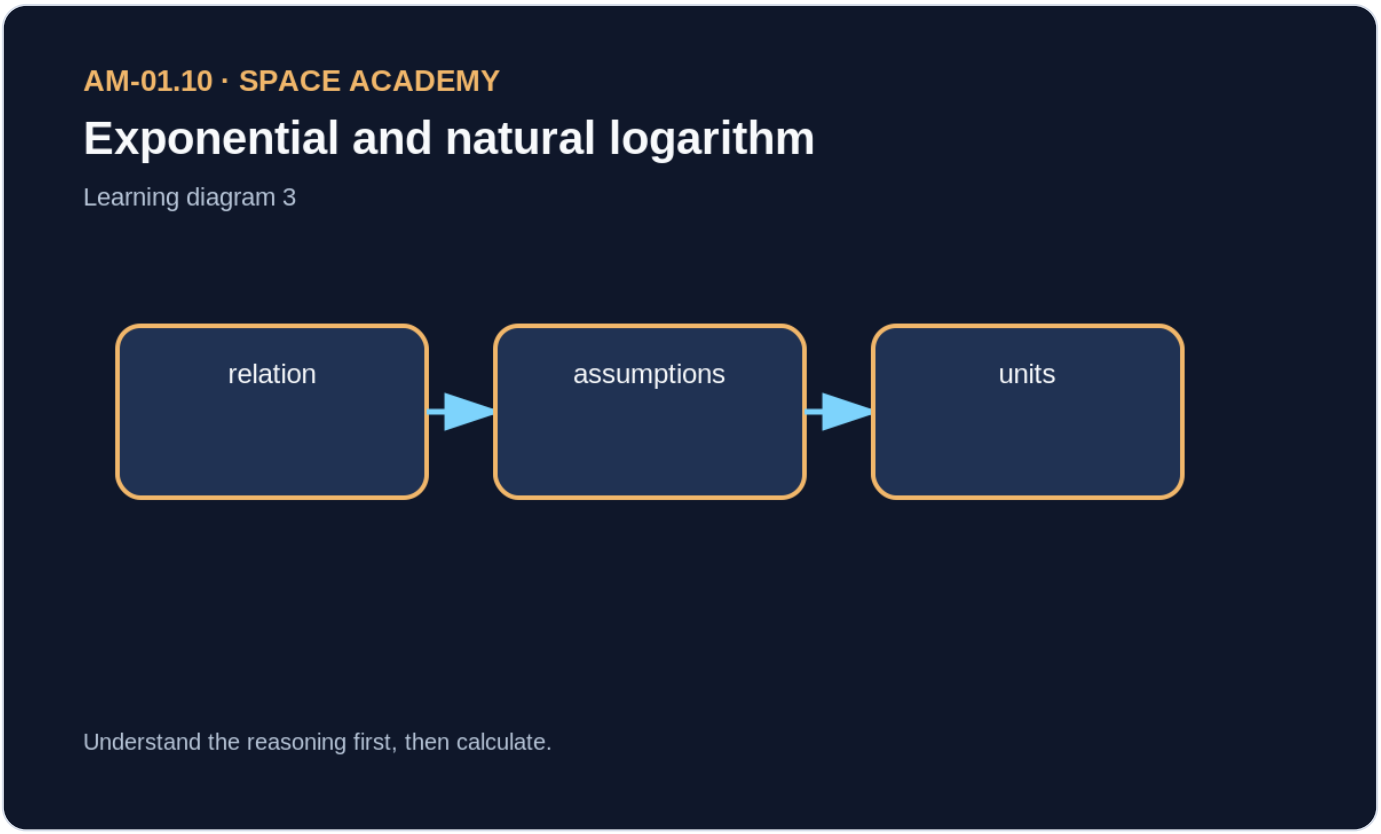
Because each step multiplies by 2. Dividing by ln(2) converts the natural-log measure into a number of doublings.

5 — Concrete example 2: why a logarithm appears in the ideal rocket equation

$$\Delta v = v_e \times \ln(m_0 \div m_1)$$

Δv is read “delta vee” and is a change in velocity. v_e is the effective exhaust velocity in this simplified model.

If $m_0/m_1=2$, $\ln(2)\approx0.693$. If the mass ratio becomes 4, $\ln(4)\approx1.386$. The benefit does not grow directly in proportion to mass ratio.



The logarithm converts a multiplicative mass ratio into an additive velocity contribution.

6 — Why must the argument of ln be dimensionless?

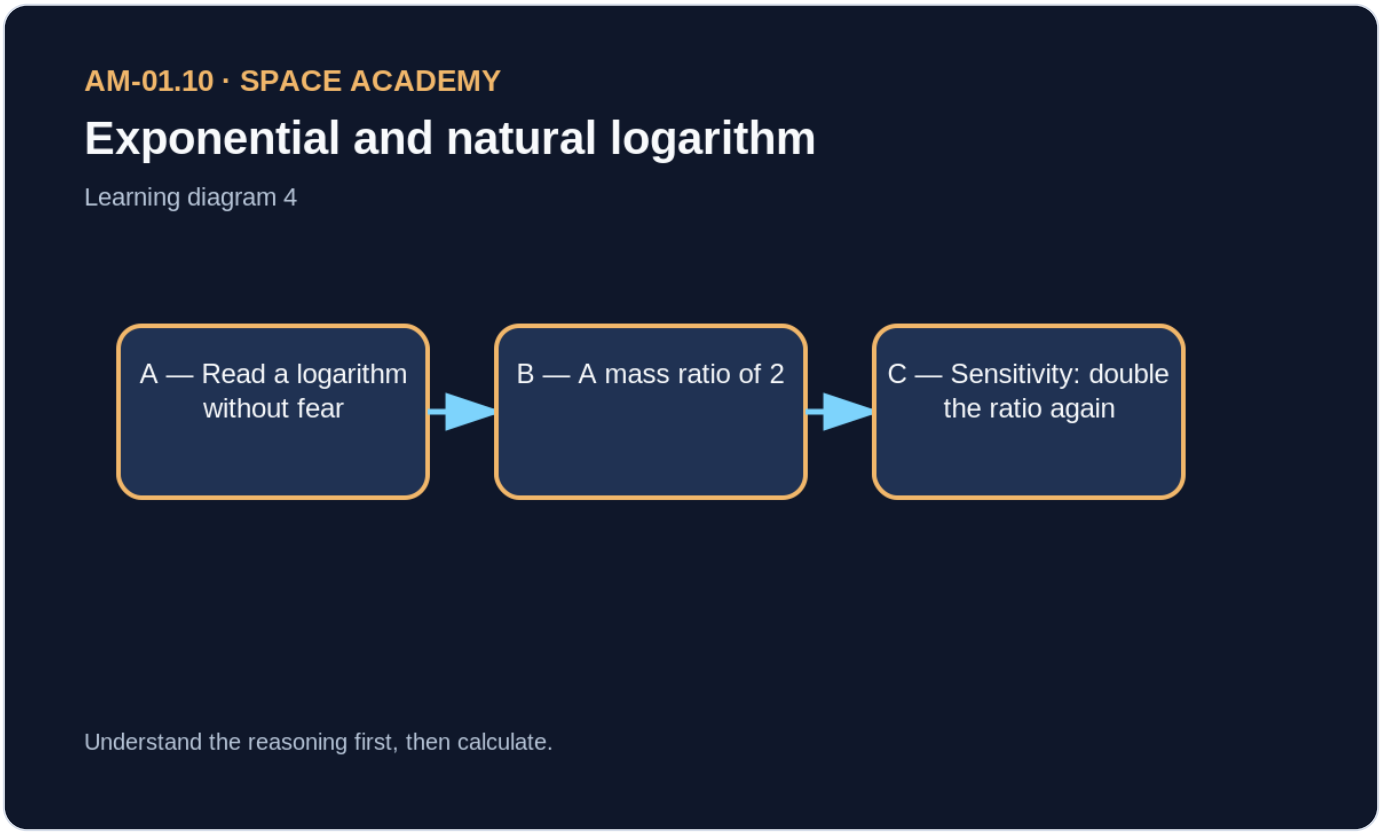
In $\ln(m_0/m_1)$, both m_0 and m_1 are masses. If both use kilograms, kg/kg cancels, leaving a dimensionless ratio.

Writing ln(500 kg) without a reference quantity is not the same kind of meaningful dimensionless operation.

7 — Calculator: ln, e^x and common mistakes

Compute ln(2): approximately 0.693. Then compute e^0.693 and you recover approximately 2.

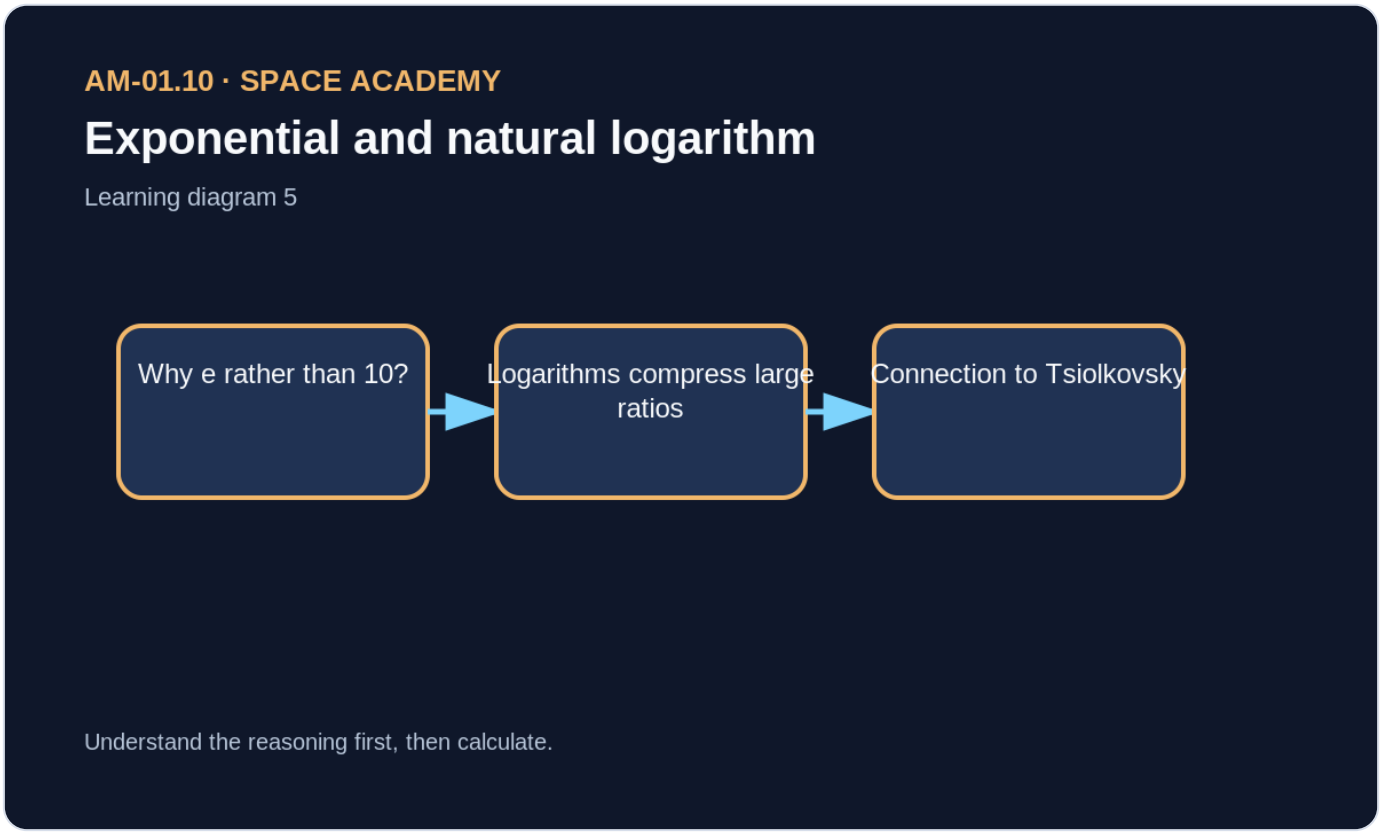
> **Do not confuse ln and $\log_{10}$.** Many calculators provide both. They are different logarithms.



ln and e^x undo each other.

8 — Three mental pictures

- **Exponential:** from exponent to multiplication factor.
- **Logarithm:** from factor back to exponent.
- **Rocket equation:** a mass ratio contributes through a logarithm, so ever larger ratios become increasingly costly.



Start from the concrete question, then choose the function that answers it.

Exercises and answers

Exercise 1

A quantity doubles four times from 1. Final value?

$2^4=16$.

Exercise 2

Compute ln(2), then e^ln(2).

$\ln(2)\approx0.693$; $e^{0.693}\approx2$.

Exercise 3

Why is m_0/m_1 dimensionless if both masses are in kg?

Because kg/kg cancels.

Primary and technical sources

- NASA Glenn — Rocket equation educational material
- NIST — SI