Course compass

Question directrice : How can a slanted direction be split into horizontal and vertical parts, and why do sine and cosine do that?

Markers: 📏 MEASURED · 📐 CONVENTION · 🧱 CALCULATED · 🎓 TEACHING ASSUMPTION · ⚠️ APPROXIMATION

- understand the concept
- do a simple calculation
- explain every symbol
- check a result

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1 — Before sine and cosine: the essential words

Start with one simple idea: **one slanted arrow can be understood as two actions happening at the same time**. One part goes left to right; the other goes bottom to top. Mathematics then lets us calculate the size of each part.

What is an angle?

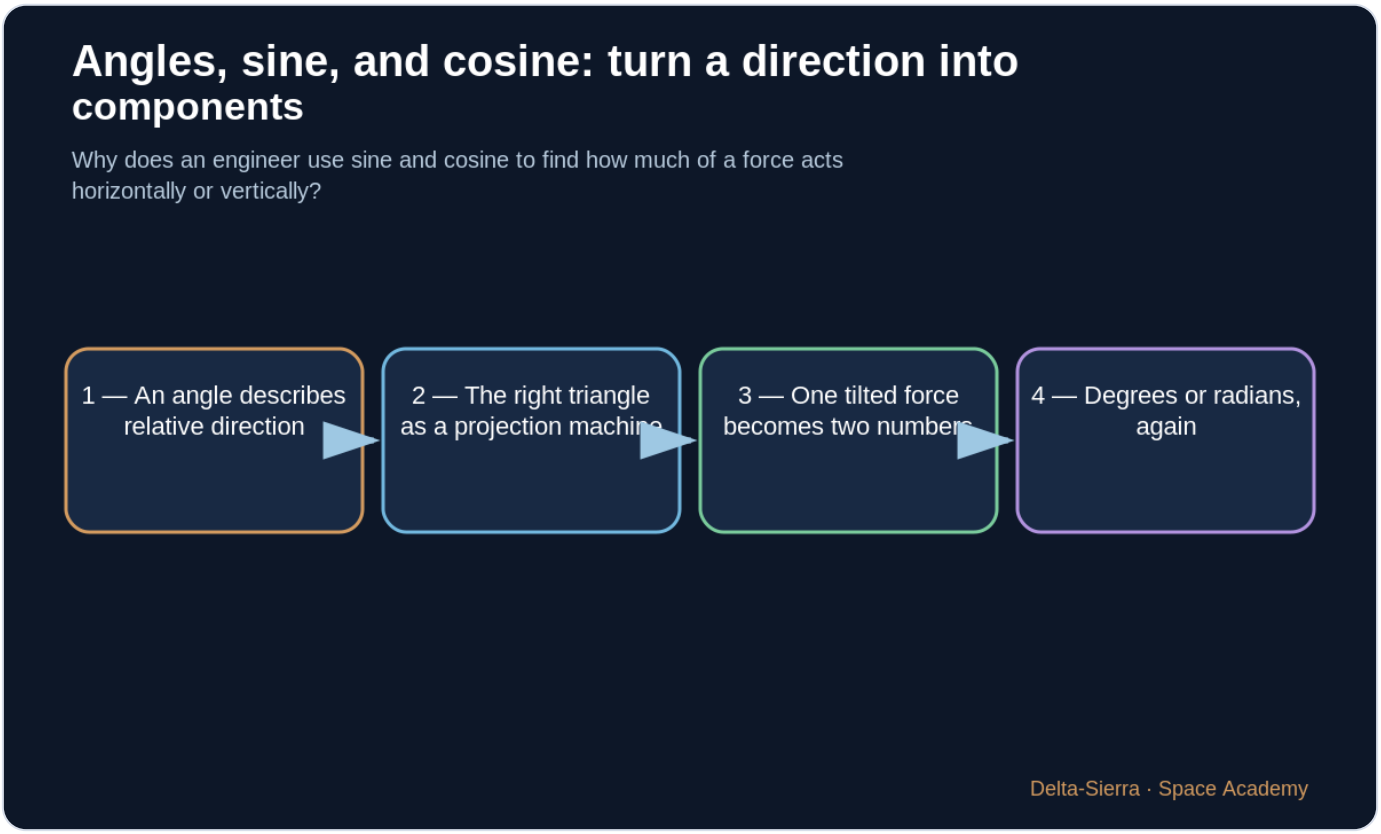
An **angle** measures the opening between two directions. Two lines shaped like the corner of a sheet of paper form a **right angle** of **90 degrees**, written 90°.

Horizontal, vertical, perpendicular

Horizontal means left to right in our chosen diagram. **Vertical** means bottom to top. Two directions are **perpendicular** when they form a 90° right angle.

What is a right triangle?

It is a triangle containing one right angle. Its longest side, opposite the right angle, is the **hypotenuse**. In this lesson that side will often represent the total slanted arrow.



Vocabulary comes before the formula: angle, horizontal, vertical, right angle and right triangle.

2 — What do “horizontal part” and “vertical part” mean?

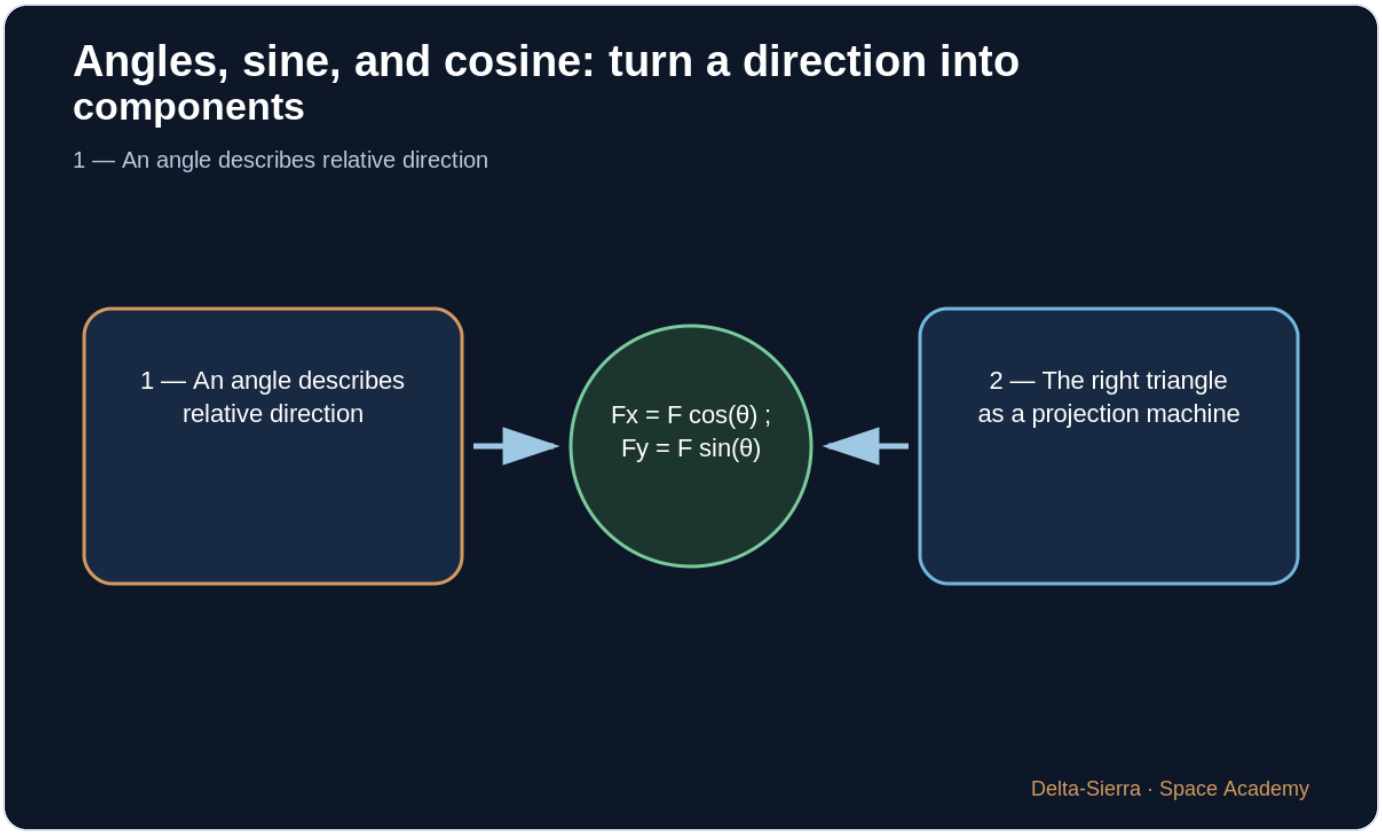
Imagine a rover climbing a slope. Between its start and end points it has done two things at once: it has moved **forward** and moved **upward**.

The **horizontal part** is the amount corresponding only to left-right motion. The **vertical part** is the amount corresponding only to climbing or descending.

These are not two unrelated actions invented afterwards. They are two simple descriptions of **the same slanted action**.

Mental picture

Draw a slanted arrow from start to finish. Now draw an L-shaped route: first horizontally until you are directly below the endpoint, then vertically to the endpoint. Those two legs reconstruct the slanted arrow.



A slanted direction can be described by one horizontal part and one vertical part.

3 — What do F, Fx, Fy, cos and sin mean?

In a force example, **F** means the **total force**: the full slanted arrow.

Fx is read “F x”. It means the horizontal part of the force along the x-axis. **Fy** is read “F y”. It means the vertical part along the y-axis.

cos(angle) is read “cosine of the angle”. **sin(angle)** is read “sine of the angle”.

The two formulas

$Fx = F \times \cos(\text{angle})$ is read: “F x equals F multiplied by the cosine of the angle.”

$Fy = F \times \sin(\text{angle})$ is read: “F y equals F multiplied by the sine of the angle.”

They turn one slanted force into two easier quantities: how much acts horizontally and how much acts vertically.

What is a newton?

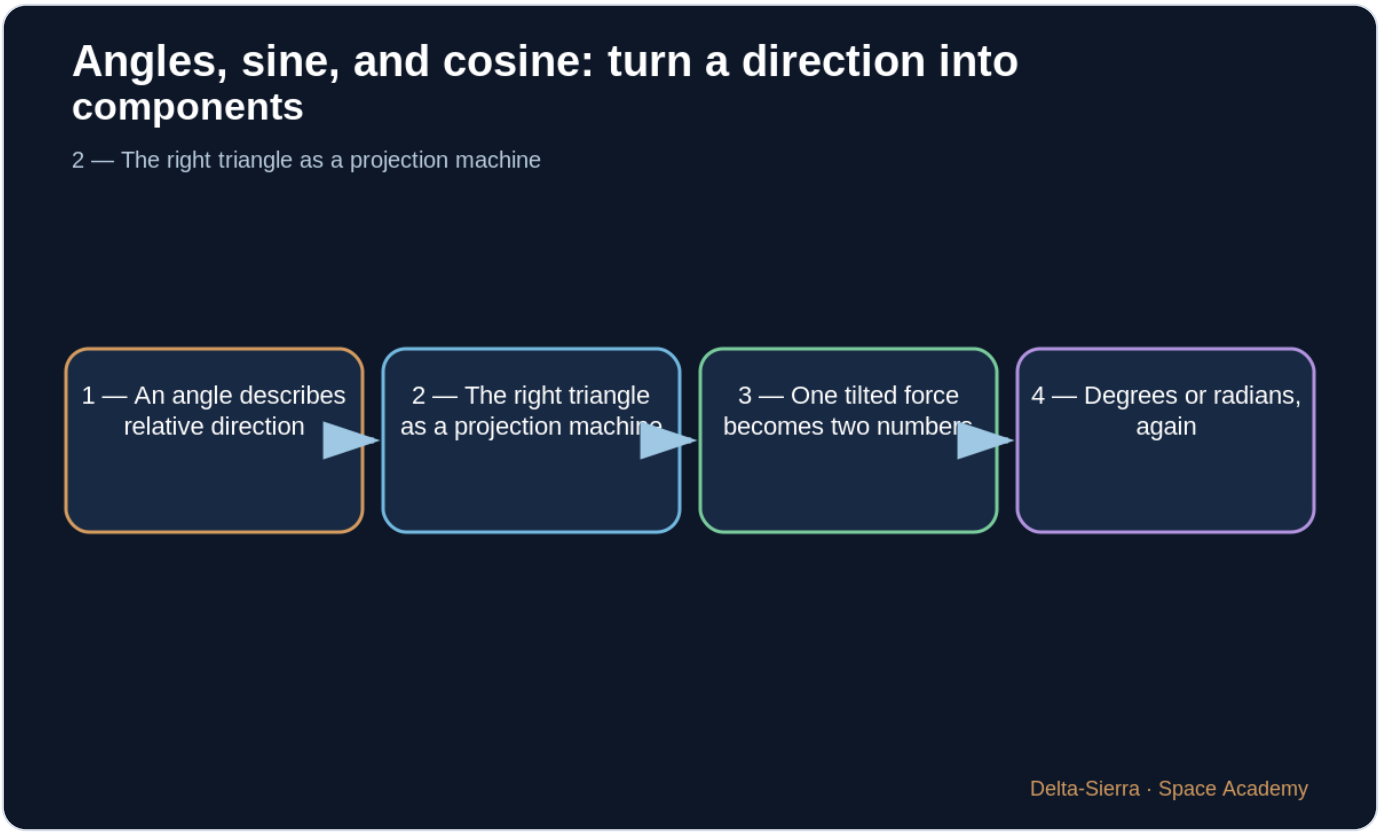
The **newton**, symbol N, is the SI unit of force. At beginner level, think of a force as a push or pull.

4 — Why does cosine give the horizontal part and sine the vertical part?

Here the angle is measured **from the horizontal**. The horizontal side touches that angle and is called the **adjacent side**. The vertical side lies across from the angle and is called the **opposite side**.

Cosine compares adjacent side with hypotenuse. Sine compares opposite side with hypotenuse. That is why, with an angle measured from the horizontal, cosine gives the horizontal part and sine gives the vertical part.

If the angle were measured from the vertical, the roles relative to the axes would swap. Always ask: **from which direction is the angle measured?**



The choice of cosine or sine depends on which side is adjacent or opposite to the chosen angle.

5 — Three concrete examples, line by line

Example A — A rover travels 10 m up a 30° slope

Total displacement: 10 m. Angle: 30° above horizontal.

$\cos(30°) \approx 0.866$; $\sin(30°)=0.5$.

Horizontal part: $10 \times 0.866 \approx 8.66\ \text{m}$.

Vertical part: $10 \times 0.5 = 5\ \text{m}$.

In plain language: the rover moves about 8.66 m horizontally and gains 5 m of height.

Example B — A lander descends at 20 m/s along a 60° path

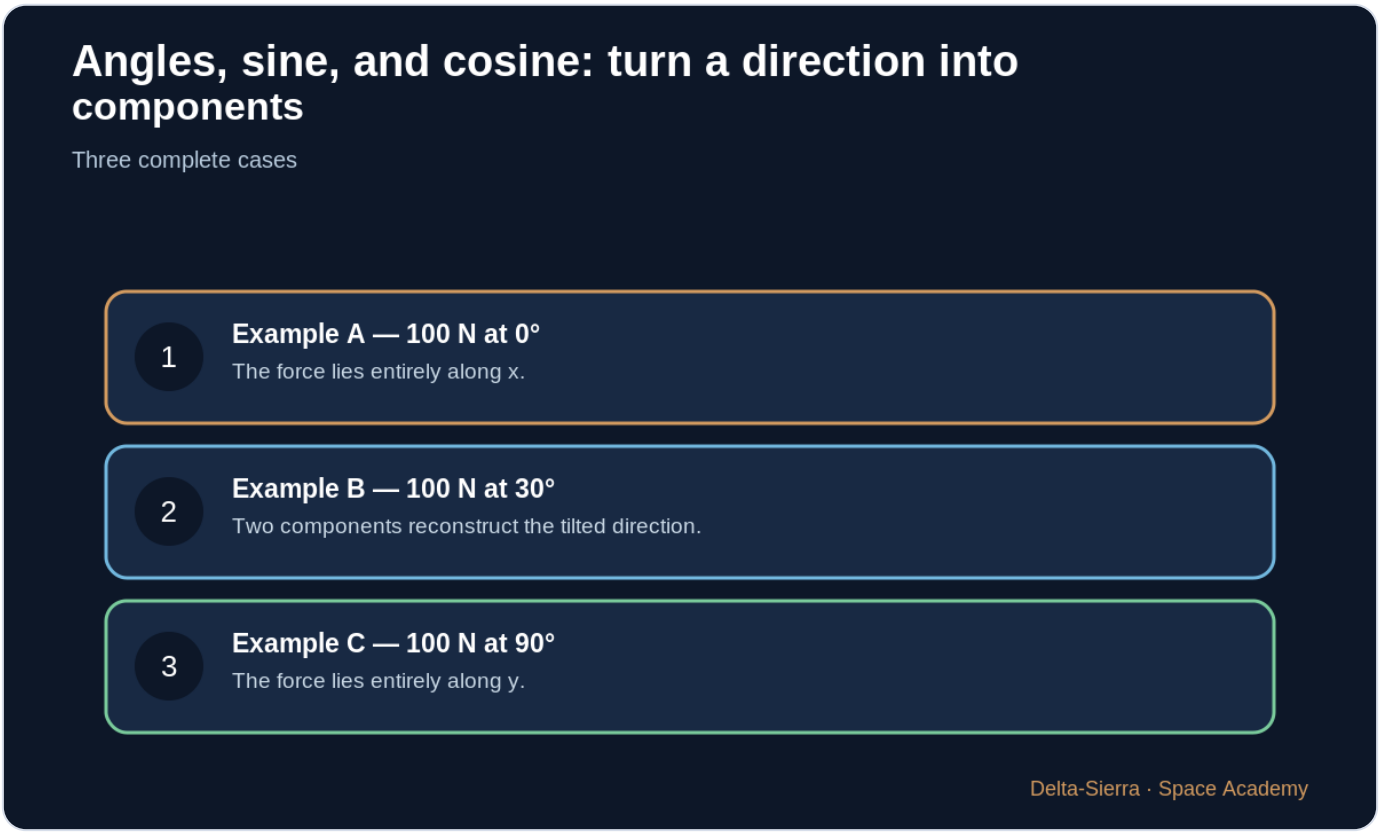
The angle is measured from horizontal. Horizontal speed: $20 \times \cos(60°)=10\ \text{m/s}$. Vertical speed magnitude: $20 \times \sin(60°)\approx 17.32\ \text{m/s}$. Because the lander is descending, that vertical component points downward.

Example C — A cable pulls with 100 N at 30°

$F_x = 100 \times \cos(30°) \approx 86.6\ \text{N}$.

$F_y = 100 \times \sin(30°) = 50\ \text{N}$.

The cable pulls forward with about 86.6 N and upward with 50 N.



The same idea works for displacement, velocity and force.

6 — Calculator: check degree mode

For these examples, the calculator must be in **DEG** mode. If it is in RAD mode, entering 30 will mean something different.

Horizontal part

Check DEG. Enter **30**, press **cos**, read about 0.8660254, multiply by **10**, obtain about 8.660254, and round sensibly to **8.66 m**.

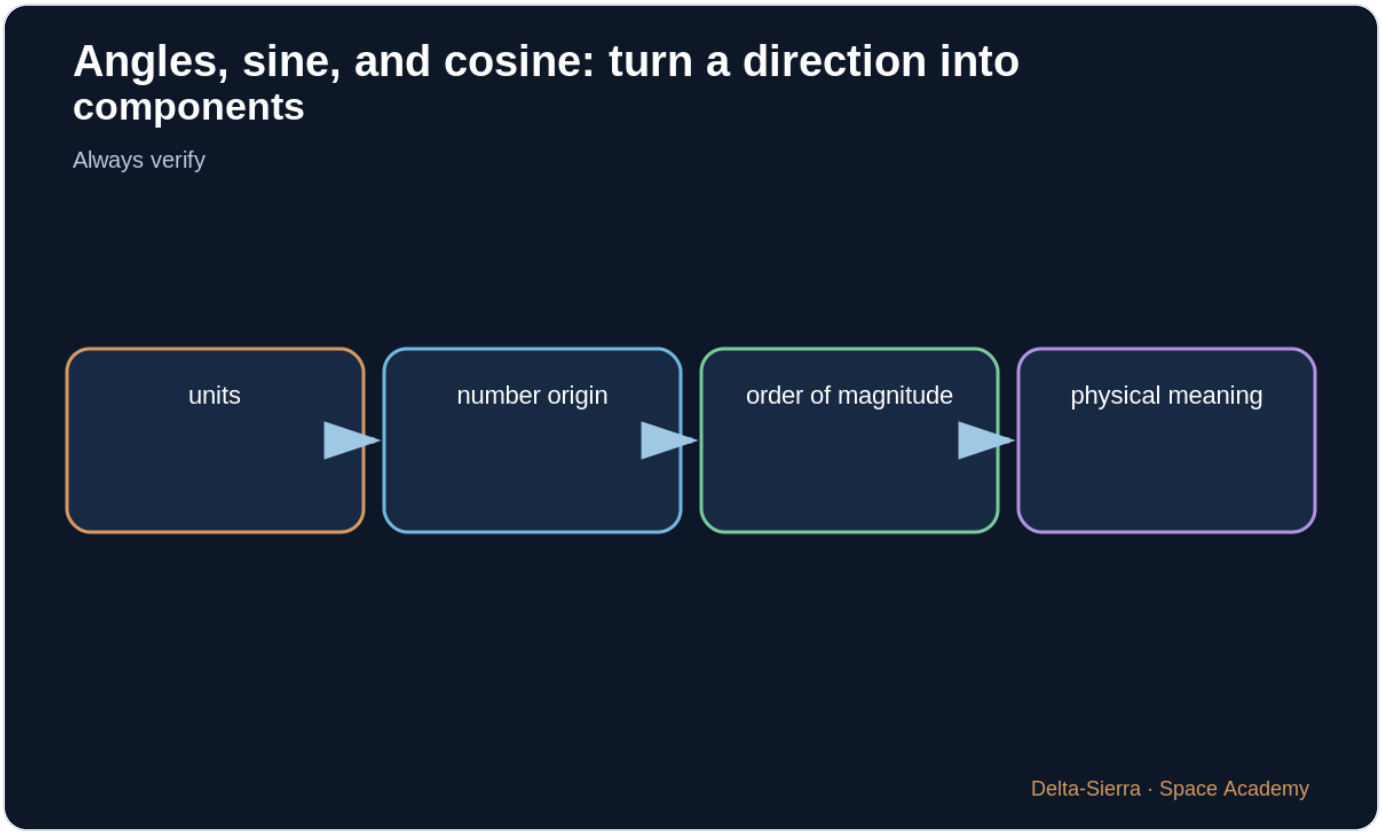
Vertical part

Enter **30**, press **sin**, read 0.5, multiply by **10**, obtain **5 m**.

7 — Three angles for sanity checks

- **0°:** fully horizontal. cos(0°)=1, sin(0°)=0.
- **45°:** horizontal and vertical components have equal magnitude, about 70.7% of the total.
- **90°:** fully vertical. cos(90°)=0, sin(90°)=1.

If a calculation gives a large horizontal component for a perfectly vertical arrow, something is wrong.



Compare the result with a simple case you can picture.

8 — Common traps

- Using a formula without knowing where the angle is measured from.
- Confusing the total arrow with one component.
- Forgetting that horizontal and vertical depend on the chosen reference frame.
- Using radians when the exercise gives degrees.
- Giving a number without its unit.

Exercises and answers

Exercise 1

A 20 m displacement is 30° above horizontal. Find its horizontal and vertical parts.

Answer: horizontal ≈ 17.32 m; vertical = 10 m.

Exercise 2

A 50 N force is perfectly vertical. What are its components?

Answer: Fx = 0 N; Fy = 50 N.

Exercise 3

Why must you know where the angle is measured from?

Answer: because adjacent and opposite sides depend on the chosen angle, so the relation to horizontal and vertical changes.

Primary and educational sources

- NIST SP 811 — Guide for the Use of the SI
- NASA Glenn — Rocket Control