AM-00.05 — Orders of magnitude: know whether a result is plausible before the decimal point

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Course compass

Guiding question: How can you catch an error by a factor of 10, 1,000, or a million without repeating the entire calculation?

Markers: MEASURED · CONVENTION · CALCULATED · TEACHING ASSUMPTION · APPROXIMATION

• understand the concept

• do a simple calculation

• explain every symbol

• check a result

Essential answer

Orders of magnitude: know whether a result is plausible before the decimal point. The question to solve is: How can you catch an error by a factor of 10, 1,000, or a million without repeating the entire calculation? Markers: MEASURED · CONVENTION · CALCULATED · TEACHING ASSUMPTION · APPROXIMATION understand the concept do a simple calculation explain every symbol check a result? Between 980 and 1,020 details differ but scale stays near 10^3. Between 1,000 and 1,000,000, scale changes by three powers of ten. The rest of the course turns that idea into an auditable line of reasoning: explicit units, stated assumptions, reproducible calculations, order-of-magnitude checks and interpretation limits. A result is useful only when the reader can explain what it measures, where every input came from and which engineering decision it can support.

Starting synthesis: derivations, examples, limitations and sources are developed in the course body.

Key terms before you start

unit · order of magnitude · assumption · approximation · power

1 — Order of magnitude looks at scale

Between 980 and 1,020 details differ but scale stays near 10^3. Between 1,000 and 1,000,000, scale changes by three powers of ten.

Space problems routinely span millimetres to millions of kilometres.

2 — Round to estimate

Estimation temporarily replaces awkward values with easy neighbours: 327 becomes about 330 and 9.81 becomes about 10.

The estimate is not the final answer; it is an independent guardrail.

3 — Prefixes: kilo, mega, giga

Kilo means one thousand, mega one million, giga one billion in SI usage. Confusing kN and N creates a factor of 1,000.

Prefixes are part of the quantity, not typography.

4 — Plausibility before precision

An engine producing a few newtons cannot lift a launch vehicle weighing hundreds of tonnes; a few seconds is not an interplanetary transfer time.

Comparing with known cases or physical bounds catches many errors.

Three complete examples: change one assumption to understand

Before each calculation, identify where every number comes from and whether it is measured, conventional, assumed, or calculated.

Example A — mental multiplication

327 × 19.8 can be estimated as 330 × 20 = 6,600. The exact result should be near 6.6×10^3, not 66 or 660,000.

APPROXIMATION: estimation gives a scale, not the final value.

Example B — kilo

1,371 kN = 1,371,000 N = 1.371 × 10^6 N.

SI CONVENTION: $k = 10^3$.

Example C — distance

225,000,000 km can be written 2.25 × 10^8 km, making the scale immediately visible.

CALCULATED: move the decimal point eight places.

Inverse calculation

If a quantity is 3 × 10^7, division by 10^3 gives 3 × 10^4. Exponents subtract because 10^7 ÷ 10^3 = 10^(73).

Common trap and result check

Trap: thinking an estimate that is 5% off is useless. To catch a ×1,000 mistake, a 5% estimate is excellent.

Always check units, order of magnitude, and physical meaning before accepting a result.

Exercises and answers

Understand

Explain the relationship in your own words.

Change an assumption

Modify one input and predict the direction of change before calculating.

Verify

Name two checks after a calculation.

Primary and educational sources

• NIST - International System of Units (SI)

• NASA Glenn - Guide to Rockets