

Useful mathematics: from proportions to derivatives



Engineering mathematics turns verbal questions into checkable relationships. The goal is not symbol memorization but model construction and assumption checking.

Every Mars example is deliberately simplified. A number calculated in an exercise does not become a mission requirement.

1. Algebra: isolate the unknown without losing the physics

An equation says two expressions represent the same quantity. If $E = P t$ and time is unknown, divide by P : $t = E/P$. Name the symbols before manipulating them: E is energy, P power, and t time. The units kWh/kW reduce to hours. [Canonical mini-lesson](#).

Exercise 1

A battery has 180 kWh usable energy and supplies 12 kW continuously. Ideal duration?

Solution: $t = 180/12 = 15$ h, before efficiency, reserve policy, and ageing. [Canonical reference](#).

Fifteen hours is an ideal duration obtained by dividing available energy by constant power. A real system then subtracts reserve, conversion losses, ageing, and auxiliary loads before promising operational endurance.

Why this matters

Engineering algebra isolates the unknown while keeping physical meaning visible. Each transformation should be reversible and units should follow the symbols.

If $E = P t$, then $t = E/P$. A battery with 90 kWh usable and a constant 6 kW load gives an ideal 15 h. The calculation below is a consistency test, not a complete truth. [Canonical mini-lesson](#).

Ideal energy endurance

$$t = E/P = 90 \text{ kWh} / 6 \text{ kW} = 15 \text{ h}$$

1 — Concrete question

What quantity must be determined in “Why this matters”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Stored energy divided by constant power gives the ideal time to depletion

3 — Quantities first

t is the ideal endurance, E is available energy, and P is average power demand.

4 — Formula

$$t = E/P = 90 \text{ kWh} / 6 \text{ kW} = 15 \text{ h}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $t = E/P = 90 \text{ kWh} / 6 \text{ kW} = 15 \text{ h}$.

6 — Symbols

t is the ideal endurance, E is available energy, and P is average power demand.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

t is in seconds or hours; E in joules, watt-hours, or kilowatt-hours; P in watts or kilowatts. Energy divided by power gives time.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “Why this matters”.

11 — Assumptions

Power must remain approximately constant and the stated energy must be genuinely usable.

12 — Unit check

t is in seconds or hours; E in joules, watt-hours, or kilowatt-hours; P in watts or kilowatts. Energy divided by power gives time. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$t = E/P = 90 \text{ kWh} / 6 \text{ kW} = 15 \text{ h}$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This duration is a first horizon before adding margins, losses and variable loads.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. A battery provides 120 kWh usable energy to a constant 8 kW load. Calculate ideal endurance.

Guided solution — open after trying

$t = E/P = 120/8 = 15$ h. kWh/kW correctly leaves hours.

Independent exercise. Calculate endurance for 72 kWh at 4.5 kW.

Independent solution — open after trying

$t = 72/4.5 = 16$ h.

21 — Mission decision

This duration is a first horizon before adding margins, losses and variable loads.

Real duration can be lower if power varies, a minimum reserve is required, or usable capacity depends on temperature.

2. Proportions: test linearity before scaling

A rule of three assumes that doubling an input doubles the output. That may approximate food mass at first order, but hospitals, power plants, and workshops have fixed costs, thresholds, and redundancy steps.

Before scaling from 4 to 100 people, separate quantities proportional to population from infrastructure that changes in discrete blocks. A standby pump or airlock does not necessarily multiply by the food factor.

Physical reading

Proportion is powerful only when the relationship is linear. Doubling population approximately doubles some consumable flows, but doubling a geometric dimension does not double volume.

At 0.4 kg/day makeup per person, 20 people need 8 kg/day and 100 people need 40 kg/day. By contrast, doubling a cylindrical tank radius quadruples its cross-sectional area.

Scale individual demand to a group

$q_{\text{total}} = N q_{\text{individual}}$; $A = \pi r^2$

1 — Concrete question

What quantity must be determined in “Physical reading”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A per-person demand becomes a group demand by multiplying by crew size; geometry then turns that demand into physical size

3 — Quantities first

q_{total} is total group demand, N is the number of people, $q_{\text{individual}}$ is demand per person, A is disk area, r is radius, and π is the dimensionless constant pi.

4 — Formula

$q_{\text{total}} = N q_{\text{individual}}$; $A = \pi r^2$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $q_{\text{total}} = N q_{\text{individual}}$; $A = \pi r^2$.

6 — Symbols

q_{total} is total group demand, N is the number of people, $q_{\text{individual}}$ is demand per person, A is disk area, r is radius, and π is the dimensionless constant pi.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

q_{total} has the group-demand unit produced by $q_{\text{individual}} \times \text{people}$. For $A = \pi r^2$, r is a length and A is an area in the corresponding squared length unit.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “Physical reading”.

11 — Assumptions

Individual demands must use the same reference period and the geometry must match the chosen shape.

12 — Unit check

q_{total} has the group-demand unit produced by $q_{\text{individual}} \times \text{people}$. For $A = \pi r^2$, r is a length and A is an area in the corresponding squared length unit. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

At $q = 0.35 \text{ kg/person/day}$ and $N = 30$, $q_{\text{total}} = 10.5 \text{ kg/day}$. For $r = 1.5 \text{ m}$, $A = \pi r^2 \approx 7.07 \text{ m}^2$.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

The chain links a human-demand assumption directly to habitat or storage sizing.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Scale $0.35 \text{ kg/person/day}$ to 30 people, then calculate the cross-section of a tank with radius 1.5 m .

Guided solution — open after trying

$q_{\text{total}} = Nq = 30 \times 0.35 = 10.5 \text{ kg/day}$. $A = \pi \times 1.5^2 \approx 7.07 \text{ m}^2$. Demand scales linearly, area quadratically.

Independent exercise. Repeat with $0.28 \text{ kg/person/day}$ for 50 people and $r = 2 \text{ m}$.

Independent solution — open after trying

$q_{\text{total}} = 50 \times 0.28 = 14 \text{ kg/day}$; $A = \pi \times 2^2 \approx 12.57 \text{ m}^2$.

21 — Mission decision

The chain links a human-demand assumption directly to habitat or storage sizing.

3. Geometry: areas, volumes, and scale laws

A disk has area $A = \pi r^2$ and a cylinder volume $V = \pi r^2 h$. Doubling radius quadruples cross-sectional area; doubling every linear dimension multiplies volume by eight. Linear intuition therefore fails quickly for tanks and habitats. [Canonical mini-lesson](#).

Area and volume of a cylinder

$$A = \pi r^2 ; V = \pi r^2 h$$

1 — Concrete question

What quantity must be determined in “3. Geometry: areas, volumes, and scale laws”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Radius sets the area of a circular section, and extending that area over a height sets volume

3 — Quantities first

A is the circular cross-sectional area, V is cylinder volume, r is radius, h is cylinder height, and π is the dimensionless constant pi.

4 — Formula

$$A = \pi r^2 ; V = \pi r^2 h$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $A = \pi r^2 ; V = \pi r^2 h$.

6 — Symbols

A is the circular cross-sectional area, V is cylinder volume, r is radius, h is cylinder height, and π is the dimensionless constant pi.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

r and h use one coherent length unit; A is in length^2 and V in length^3 . For metres, A is m^2 and V is m^3 .

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

This operation is used because the relationship directly represents the dependency studied in “3. Geometry: areas, volumes, and scale laws”.

11 — Assumptions

The model assumes cylindrical geometry and dimensions measured in one unit system.

12 — Unit check

r and h use one coherent length unit; A is in length^2 and V in length^3 . For metres, A is m^2 and V is m^3 . The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

For $r = 1.2 \text{ m}$ and $h = 2.5 \text{ m}$, $A = \pi \times 1.2^2 \approx 4.52 \text{ m}^2$ and $V = A \times 2.5 \approx 11.31 \text{ m}^3$.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

A second calculation path or inverse operation should recover the original quantity.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

These relations turn dimensions into useful area, pressurized volume or storage capacity.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Calculate base area and volume for a cylinder with $r = 1.2$ m and $h = 2.5$ m.

Guided solution — open after trying

$$A = \pi r^2 = \pi \times 1.44 \approx 4.52 \text{ m}^2. V = Ah \approx 4.52 \times 2.5 = 11.31 \text{ m}^3.$$

Independent exercise. Repeat for $r = 0.8$ m and $h = 3$ m.

Independent solution — open after trying

$$A = \pi \times 0.8^2 \approx 2.01 \text{ m}^2; V \approx 2.01 \times 3 = 6.03 \text{ m}^3.$$

21 — Mission decision

These relations turn dimensions into useful area, pressurized volume or storage capacity.

Exercise 3

Cylindrical tank: $r = 1.2$ m and $h = 3.0$ m. Find geometric volume. [Canonical reference.](#)

Solution: $\pi \times 1.2^2 \times 3 \approx 13.57 \text{ m}^3$. Usable volume is lower after structure, hardware, and ullage. [Canonical reference.](#)

The geometric calculation gives only the envelope. Real usable volume is reduced by walls, internal structure, sensors, gas space, and operating margins; that distinction belongs in the inventory model.

Case study — why volume grows faster than length

Consider a cylindrical tank with radius 1.5 m and height 4 m. Its volume is $\pi r^2 h \approx 28.27 \text{ m}^3$. If every linear dimension doubles, radius becomes 3 m and height 8 m; volume becomes about 226.2 m^3 , eight times larger. External area grows only by a factor of four. This difference is why scaling changes structural, insulation, and storage balances. [Canonical reference.](#)

Doubling only the height while keeping radius fixed doubles the volume. The phrase “twice as large” is therefore not a mathematical specification. State which dimension changes. On Mars this affects wall mass, heated area, pressurized volume, and transfer time. Geometry tells you which quantities grow with L , L^2 , or L^3 . [Canonical reference.](#)

Volume scaling law

$$V = \pi r^2 h = \pi \times 1.5^2 \times 4 \approx 28.27 \text{ m}^3; \text{ all lengths } \times 2 \Rightarrow V \times 8$$

1 — Concrete question

What quantity must be determined in “Exercise 3”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

When all lengths are scaled by the same factor, volume changes with the cube of that factor

3 — Quantities first

V is cylinder volume, r is radius, h is height, and π is the dimensionless constant pi. The scaling statement compares geometrically similar cylinders.

4 — Formula

$$V = \pi r^2 h = \pi \times 1.5^2 \times 4 \approx 28.27 \text{ m}^3; \text{ all lengths } \times 2 \Rightarrow V \times 8$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $V = \pi r^2 h = \pi \times 1.5^2 \times 4 \approx 28.27 \text{ m}^3$; all lengths $\times 2 \Rightarrow V \times 8$.

6 — Symbols

V is cylinder volume, r is radius, h is height, and π is the dimensionless constant pi. The scaling statement compares geometrically similar cylinders.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

If r and h are in metres, V is in m^3 . Doubling every length multiplies a three-dimensional volume by $2^3=8$.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The mathematical choice is direct: the relationship directly represents the dependency studied in “Exercise 3”.

11 — Assumptions

Geometric similarity must be preserved; changing only one dimension does not follow the same scaling law.

12 — Unit check

If r and h are in metres, V is in m^3 . Doubling every length multiplies a three-dimensional volume by $2^3=8$. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$V = \pi r^2 h = \pi \times 1.5^2 \times 4 \approx 28.27 \text{ m}^3$; all lengths $\times 2 \Rightarrow V \times 8$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

Working backward with the inverse operation provides an independent check.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This law prevents severe underestimation of mass, volume and demand when a system is scaled up.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. A cylinder has $r = 2 \text{ m}$ and $h = 5 \text{ m}$. Calculate V, then the volume if all lengths double.

Guided solution — open after trying

$V = \pi \times 2^2 \times 5 = 20\pi \approx 62.83 \text{ m}^3$. After $\times 2$ scaling: $r=4$, $h=10$, $V=160\pi \approx 502.65 \text{ m}^3$, exactly 8 times larger.

Independent exercise. What happens if all lengths are multiplied by 1.5?

Independent solution — open after trying

Volume scales by $1.5^3 = 3.375$. An initial 62.83 m^3 becomes $\approx 212.05 \text{ m}^3$.

21 — Mission decision

This law prevents severe underestimation of mass, volume and demand when a system is scaled up.

Move into calculation

Geometry becomes a resource problem when a habitat must be pressurized, heated, shielded, or manufactured. Habitable volume, wall area, and shielding mass do not scale the same way.

A cylinder of radius 3 m and length 8 m provides $V \approx \pi \times 9 \times 8 \approx 226 \text{ m}^3$ before internal fittings. A uniform 2 m regolith layer over 100 m^2 represents 200 m^3 of material before porosity. The teaching value is seeing units turn into a decision.

Cylinder and layer volumes

$$V_{\text{cyl}} = \pi r^2 L ; V_{\text{layer}} = A e$$

1 — Concrete question

What quantity must be determined in “Move into calculation”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Two simple geometries cover many problems: a cylinder for a tank or duct, and area times thickness for a layer

3 — Quantities first

V_{cyl} is cylinder volume, r is radius, L is cylinder length, V_{layer} is layer volume, A is covered area, e is layer thickness, and π is the dimensionless constant pi.

4 — Formula

$$V_{\text{cyl}} = \pi r^2 L ; V_{\text{layer}} = A e$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $V_{\text{cyl}} = \pi r^2 L ; V_{\text{layer}} = A e$.

6 — Symbols

V_{cyl} is cylinder volume, r is radius, L is cylinder length, V_{layer} is layer volume, A is covered area, e is layer thickness, and π is the dimensionless constant pi.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

V_{cyl} and V_{layer} are in volume units such as m^3 ; r , L , and e are lengths such as m ; A is area such as m^2 ; π is dimensionless.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The mathematical choice is direct: the relationship directly represents the dependency studied in “Move into calculation”.

11 — Assumptions

The real shape must be close enough to the chosen geometric model.

12 — Unit check

V_{cyl} and V_{layer} are in volume units such as m^3 ; r , L , and e are lengths such as m ; A is area such as m^2 ; π is dimensionless. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

With $r = 2 \text{ m}$ and $L = 6 \text{ m}$, $V_{\text{cyl}} = \pi \times 2^2 \times 6 \approx 75.40 \text{ m}^3$. A layer $A = 40 \text{ m}^2$, $e = 0.25 \text{ m}$ gives $V_{\text{layer}} = 10 \text{ m}^3$.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

Working backward with the inverse operation provides an independent check.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

These approximations quickly give orders of magnitude for material, excavation or capacity.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

- Vary one input at a time to identify which assumption drives the result most strongly.
- 20 — Guided and autonomous practice
- Guided exercise. Calculate a cylinder $r = 2$ m, $L = 6$ m and a 0.25 m layer over 40 m².

Guided solution — open after trying

$$V_{\text{cyl}} = \pi r^2 L = \pi \times 4 \times 6 = 24\pi \approx 75.40 \text{ m}^3. \quad V_{\text{layer}} = Ae = 40 \times 0.25 = 10 \text{ m}^3.$$

Independent exercise. Repeat with $r = 1.5$ m, $L = 8$ m and a 0.40 m layer over 30 m².

Independent solution — open after trying

$$V_{\text{cyl}} = \pi \times 1.5^2 \times 8 = 18\pi \approx 56.55 \text{ m}^3; \quad V_{\text{layer}} = 30 \times 0.40 = 12 \text{ m}^3.$$

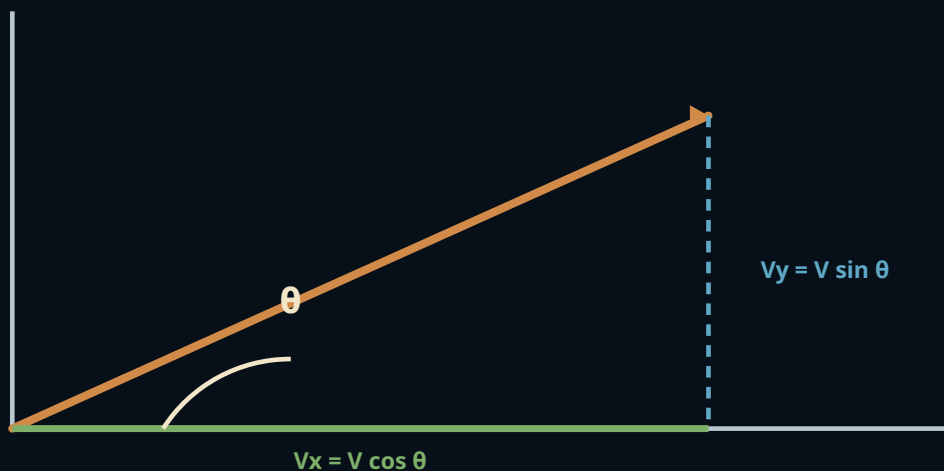
- 21 — Mission decision

These approximations quickly give orders of magnitude for material, excavation or capacity.

A geometrically efficient shape can be difficult to build, inspect, or repair; the mathematical optimum is not automatically the industrial optimum.

Space Academy · 01

Vecteur et composantes / vector components



Direction is part of the quantity

Vector resolution: magnitude and angle become measurable components before recombination.

4. Trigonometry: turn an angle into components

For vector magnitude V at angle θ above horizontal, $V_x = V \cos \theta$ and $V_y = V \sin \theta$. The same decomposition appears in slopes, robotics, tilted thrust, and line of sight. [Canonical mini-lesson](#).

Check degree versus radian mode. A simple sanity check is that each orthogonal component must not exceed the original magnitude.

Exercise 4

$V = 10$ m/s at 30° . Find V_x and V_y . [Canonical reference](#).

Solution: $V_x \approx 8.66$ m/s and $V_y = 5.00$ m/s. [Canonical mini-lesson](#).

Control point

Trigonometry turns angles and lengths into measurable components. On Mars it appears in slopes, lines of sight, antennas, forces, and local trajectories.

A 10° slope over 100 m horizontal distance corresponds to about 17.6 m elevation change because $\tan(10^\circ) \approx 0.176$. An engineer keeps numerical result, margin, and validity domain separate. [Canonical reference](#).

Elevation change from slope

$$\Delta z = d \tan \theta = 100 \text{ m} \times \tan 10^\circ \approx 17.6 \text{ m}$$

1 — Concrete question

What quantity must be determined in “Control point”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A horizontal distance and slope angle determine the corresponding elevation change

3 — Quantities first

Δz is vertical elevation change, d is the horizontal or path distance used by the model, and θ is the slope angle.

4 — Formula

$$\Delta z = d \tan \theta = 100 \text{ m} \times \tan 10^\circ \approx 17.6 \text{ m}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $\Delta z = d \tan \theta = 100 \text{ m} \times \tan 10^\circ \approx 17.6 \text{ m}$.

6 — Symbols

Δz is vertical elevation change, d is the horizontal or path distance used by the model, and θ is the slope angle.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

Δz and d use the same length unit, such as m; $\tan \theta$ is dimensionless. θ may be entered in degrees or radians according to calculator convention.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

Here the operation expresses the following mechanism: the relationship directly represents the dependency studied in “Control point”.

11 — Assumptions

The angle must be defined relative to horizontal and the approximation must fit the terrain geometry.

12 — Unit check

Δz and d use the same length unit, such as m; $\tan \theta$ is dimensionless. θ may be entered in degrees or radians according to calculator convention. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$\Delta z = d \tan \theta = 100 \text{ m} \times \tan 10^\circ \approx 17.6 \text{ m}$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An inverse calculation should reconstruct one input within rounding error.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This calculation turns slope into an access, grading or mobility constraint.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Calculate elevation change for 250 m horizontal at 6°.

Guided solution — open after trying

$$\Delta z = d \tan \theta = 250 \times \tan 6^\circ \approx 250 \times 0.1051 = 26.28 \text{ m.}$$

Independent exercise. Calculate elevation change for 80 m at 12°.

Independent solution — open after trying

$$\Delta z = 80 \times \tan 12^\circ \approx 80 \times 0.2126 = 17.01 \text{ m.}$$

21 — Mission decision

This calculation turns slope into an access, grading or mobility constraint.

Check whether the stated distance is horizontal, along the slope, or slant range; the same angle then leads to different equations.

5. Vectors: direction is part of the data

Mass is scalar. Force, velocity, and acceleration are vectors. Two 5 m/s velocities, one east and one north, combine to magnitude $\sqrt{(5^2+5^2)} \approx 7.07$ m/s, not 10 m/s. [Canonical mini-lesson](#).

Vector components also combine pointing errors and forces. Choosing clear axes turns one spatial problem into several scalar equations and then reconstructs the direction.

Case study — add displacement vectors, not just distances

A rover travels 8 km east and then 6 km north. Path length is 14 km, while straight-line displacement is $\sqrt{(8^2+6^2)}=10$ km. Its direction relative to east is $\arctan(6/8)\approx 36.9^\circ$. This elementary distinction becomes essential for navigation, velocity vectors, and relative wind. [Canonical mini-lesson](#). [Canonical reference](#).

If the rover must return to its start, the direct-return quantity is the opposite displacement vector, not the historical sum of path segments. If terrain forces a detour around a crater, actual path length again exceeds displacement magnitude. An autonomy model must say whether it uses ideal geometry, a traversable route, or accumulated distance. Mathematics keeps these objects distinct instead of hiding them inside one word. [Canonical mini-lesson](#).

Magnitude of a 2D displacement

$$||d|| = \sqrt{(8^2+6^2)} = 10 \text{ km; } \theta = \arctan(6/8) \approx 36.9^\circ$$

1 — Concrete question

What quantity must be determined in “5. Vectors: direction is part of the data”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Two perpendicular displacements form a right triangle; the direct distance is its hypotenuse

3 — Quantities first

$||d||$ is the magnitude of the net displacement vector; the two numerical terms are perpendicular displacement components; θ is the direction angle measured from the chosen horizontal axis.

4 — Formula

$$||d|| = \sqrt{8^2 + 6^2} = 10 \text{ km}; \theta = \arctan(6/8) \approx 36.9^\circ$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $||d|| = \sqrt{8^2 + 6^2} = 10 \text{ km}; \theta = \arctan(6/8) \approx 36.9^\circ$.

6 — Symbols

$||d||$ is the magnitude of the net displacement vector; the two numerical terms are perpendicular displacement components; θ is the direction angle measured from the chosen horizontal axis.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

The components and $||d||$ use the same length unit, here km. Squaring gives km^2 , addition remains km^2 , and the square root returns km. θ is an angle in degrees here.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in "5. Vectors: direction is part of the data".

11 — Assumptions

Both components must belong to the same coordinate frame and be perpendicular.

12 — Unit check

The components and $||d||$ use the same length unit, here km. Squaring gives km^2 , addition remains km^2 , and the square root returns km. θ is an angle in degrees here. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$||d|| = \sqrt{8^2 + 6^2} = 10 \text{ km}; \theta = \arctan(6/8) \approx 36.9^\circ$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

Magnitude converts navigation components into straight-line distance.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. A rover moves 9 km east and 12 km north. Calculate net displacement.

Guided solution — open after trying

$||d|| = \sqrt{9^2 + 12^2} = \sqrt{225} = 15 \text{ km}$. Travelled path in two segments is 21 km, distinct from net displacement.

Independent exercise. Calculate the magnitude for 5 km and 12 km.

Independent solution — open after trying

$$\sqrt{(5^2+12^2)} = \sqrt{169} = 13 \text{ km.}$$

21 — Mission decision

Magnitude converts navigation components into straight-line distance.

Interpretation

A vector has magnitude and direction. Adding velocities, forces, or displacements as ordinary numbers ignores orientation and can produce a wrong navigation decision.

A rover drives 3 km east and then 4 km north: path length is 7 km, while net displacement is 5 km by Pythagoras. The formula is useful because it makes a system dependency visible.

Magnitude of a position vector

$$|r| = \sqrt{(3^2+4^2)} = 5 \text{ km}$$

1 — Concrete question

What quantity must be determined in “Interpretation”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

The length of a position vector summarizes perpendicular coordinate components into one distance

3 — Quantities first

$|r|$ is the magnitude of the position vector r ; the two numerical terms are perpendicular position components.

4 — Formula

$$|r| = \sqrt{(3^2+4^2)} = 5 \text{ km}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $|r| = \sqrt{(3^2+4^2)} = 5 \text{ km}$.

6 — Symbols

$|r|$ is the magnitude of the position vector r ; the two numerical terms are perpendicular position components.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

Both components use the same length unit, here km. Squaring gives km^2 and the square root returns km for $|r|$.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

Here the operation expresses the following mechanism: the relationship directly represents the dependency studied in “Interpretation”.

11 — Assumptions

Components must use the same frame and units.

12 — Unit check

Both components use the same length unit, here km. Squaring gives km^2 and the square root returns km for $|r|$. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$|r| = \sqrt{(3^2+4^2)} = 5 \text{ km}$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An inverse calculation should reconstruct one input within rounding error.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

Magnitude gives distance from the origin but not direction or a traversable route.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Calculate the magnitude of position vector (6,8) km.

Guided solution — open after trying

$$|r| = \sqrt{(36+64)} = \sqrt{100} = 10 \text{ km.}$$

Independent exercise. Calculate $|r|$ for (7,24) km.

Independent solution — open after trying

$$\sqrt{(49+576)} = \sqrt{625} = 25 \text{ km.}$$

21 — Mission decision

Magnitude gives distance from the origin but not direction or a traversable route.

Displacement does not determine energy use: terrain, slope, and speed can make a geometrically shorter route more expensive.

6. Functions and graphs: see trends

A function maps input to output. Power versus time reveals peaks; CO₂ versus time reveals drift and control lag; efficiency versus load reveals operating regions.

Always inspect axes, units, origin, and scale. A truncated axis can exaggerate change. A logarithmic axis represents ratios rather than equal absolute steps.

Field reasoning

A function links an input to an output. A graph exposes thresholds, saturation, drift, and regions where a linear approximation stops being acceptable.

If a battery provides 100 kWh usable at 20°C but only 85 kWh in a cold scenario, capacity is not a universal constant; it depends on state. A model can write $E_{\text{usable}} = f(T, \text{age}, \text{power})$. [Canonical mini-lesson](#). [Canonical reference](#).

Function and dependence between variables

$$y = f(x) ; E_{\text{useful}} = f(T, \text{state}, \text{power})$$

1 — Concrete question

What quantity must be determined in “Field reasoning”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A function states that an output quantity depends on one or more inputs according to a defined rule

3 — Quantities first

y is a function output, x is its input, and f is the rule relating them. E_{useful} is useful energy; T is temperature; state describes the system state; power is the power condition supplied to the model.

4 — Formula

$y=f(x)$; $E_{\text{useful}}=f(T, \text{state}, \text{power})$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $y=f(x)$; $E_{\text{useful}}=f(T, \text{state}, \text{power})$.

6 — Symbols

y is a function output, x is its input, and f is the rule relating them. E_{useful} is useful energy; T is temperature; state describes the system state; power is the power condition supplied to the model.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

y and x keep the units defined by the specific function. E_{useful} may be in J, Wh, or kWh; T in K or °C as appropriate; power in W or kW; state may be categorical or dimensionless.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

This operation is used because the relationship directly represents the dependency studied in “Field reasoning”.

11 — Assumptions

The dependency rule and its validity domain must be known before extrapolation.

12 — Unit check

y and x keep the units defined by the specific function. E_{useful} may be in J, Wh, or kWh; T in K or °C as appropriate; power in W or kW; state may be categorical or dimensionless. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

If $f(x)=2x+5$, then $f(7)=19$. For $E_{\text{useful}}(T)=100-0.5(20-T)$, $E_{\text{useful}}(-10)=85$ kWh.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

A second calculation path or inverse operation should recover the original quantity.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This notation helps distinguish controlled variables, observed variables and calculated outcomes.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. For $f(x)=2x+5$, calculate $f(7)$, then interpret it as an input-output dependency.

Guided solution — open after trying

$f(7)=2\times 7+5=19$. The function applies one rule: input 7 maps to output 19.

Independent exercise. With $E_{\text{useful}}(T)=100-0.5(20-T)$, calculate E_{useful} at -10 °C.

Independent solution — open after trying

$20 - (-10) = 30$; $0.5 \times 30 = 15$; $E_{\text{useful}} = 100 - 15 = 85$ kWh. The model explicitly depends on temperature.

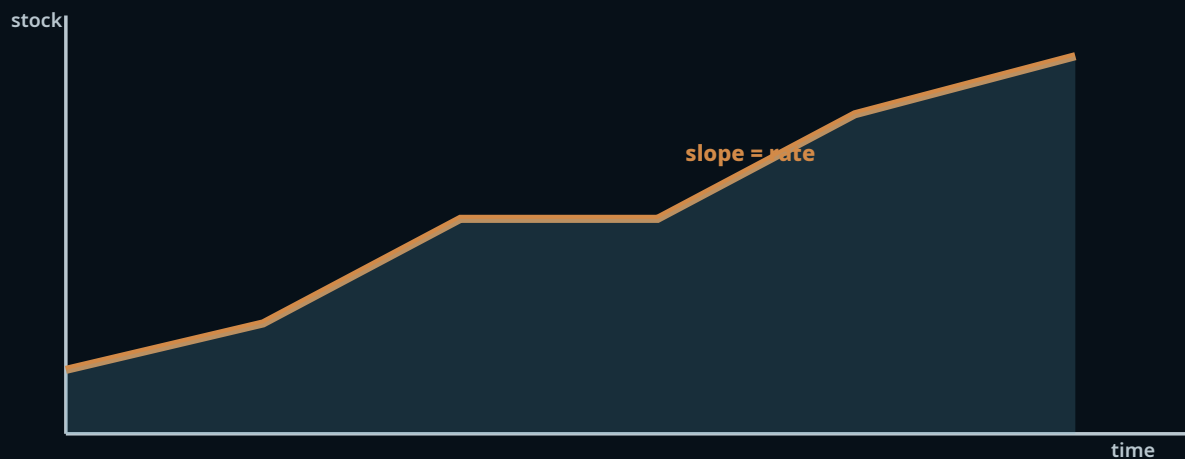
21 — Mission decision

This notation helps distinguish controlled variables, observed variables and calculated outcomes.

Drawing a line through two measured points does not justify indefinite extrapolation outside the observed domain.

Space Academy · 01

Taux et accumulation / rate and accumulation



Slope describes rate; area or sum describes accumulation

From rate to inventory: derivative, integration and balance for an accumulating quantity.

7. Derivatives: follow rates of change

If $x(t)$ is position, dx/dt is velocity and d^2x/dt^2 acceleration. If $M(t)$ is tank mass, dM/dt is net flow. A derivative asks how quickly a quantity is changing at a particular point. [Canonical reference](#).

For sampled data, $\Delta M/\Delta t$ is already a useful average slope. Rate thinking matters when a leak accelerates, temperature drift steepens, or repair time grows unexpectedly. [Canonical reference](#).

Why this matters

A derivative measures an instantaneous rate of change. It distinguishes an acceptable inventory that is deteriorating rapidly from a low but stable inventory.

If a tank drops from 1,000 to 940 kg in 6 h, the average rate is -10 kg/h. If it persisted, 240 kg would be lost in 24 h; the slope reveals the problem before the tank is empty. The calculation below is a consistency test, not a complete truth.

Average rate of change

$$dM/dt \approx \Delta M/\Delta t = -60 \text{ kg} / 6 \text{ h} = -10 \text{ kg/h}$$

1 — Concrete question

What quantity must be determined in “Why this matters”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A change divided by its time interval gives the average rate at which a stock or quantity changes

3 — Quantities first

dM/dt is the average or local mass-change rate, ΔM is the mass change over the interval, and Δt is the elapsed time.

4 — Formula

$$dM/dt \approx \Delta M/\Delta t = -60 \text{ kg} / 6 \text{ h} = -10 \text{ kg/h}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $dM/dt \approx \Delta M/\Delta t = -60 \text{ kg} / 6 \text{ h} = -10 \text{ kg/h}$.

6 — Symbols

dM/dt is the average or local mass-change rate, ΔM is the mass change over the interval, and Δt is the elapsed time.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

dM/dt is in mass/time such as kg/h; ΔM is in kg; Δt is in h or another coherent time unit. $\text{kg} \div \text{h}$ gives kg/h.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

Here the operation expresses the following mechanism: the relationship directly represents the dependency studied in “Why this matters”.

11 — Assumptions

The chosen interval must be relevant; an average rate can hide faster variations inside it.

12 — Unit check

dM/dt is in mass/time such as kg/h; ΔM is in kg; Δt is in h or another coherent time unit. $\text{kg} \div \text{h}$ gives kg/h. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$dM/dt \approx \Delta M/\Delta t = -60 \text{ kg} / 6 \text{ h} = -10 \text{ kg/h}$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An inverse calculation should reconstruct one input within rounding error.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This rate turns a trend into a loss, production or drift rate.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. A stock drops from 1,200 to 1,110 kg in 9 h. Calculate average rate.

Guided solution — open after trying

$$\Delta M = 1,110 - 1,200 = -90 \text{ kg}; \Delta t = 9 \text{ h}; \Delta M/\Delta t = -10 \text{ kg/h}.$$

Independent exercise. Another stock drops from 800 to 740 kg in 3 h. Calculate the rate.

Independent solution — open after trying

$$\Delta M = -60 \text{ kg}; \text{ rate} = -60/3 = -20 \text{ kg/h.}$$

21 — Mission decision

This rate turns a trend into a loss, production or drift rate.

A six-hour average can hide an intermittent leak; sensor time resolution is part of the model.

8. Integrals: accumulate flow or power

Integration accumulates a distributed quantity. Energy is the integral of power over time; transferred mass is the integral of mass flow. With constant values, this becomes quantity = rate $\times$ time. [Canonical reference](#).

Exercise 8

2 kW for 3 h, then 5 kW for 1 h. Energy?

Solution: $2 \times 3 + 5 \times 1 = 11$ kWh, a piecewise integration. [Canonical reference](#).

Case study — integrate variable power

Electrical load is rarely perfectly constant. Suppose a heater draws 1 kW for 2 h, 3 kW for 4 h, and 0.5 kW for 6 h. Energy is the area under the power-time curve: $1 \times 2 + 3 \times 4 + 0.5 \times 6 = 17$ kWh. When values are piecewise constant, this sum is a discrete integral. [Canonical reference](#).

With one-minute samples you can sum $P_i \Delta t$. With a continuous function $P(t)$, write $E = \int P(t) dt$. The notation changes, not the physical meaning: a rate is accumulated through time. The same idea accumulates water flow, oxygen production, or dose. Units are a built-in check: kW $\times$ h gives kWh and kg/h $\times$ h gives kg. An integral ending in the wrong unit signals a badly posed model. [Canonical mini-lesson](#). [Canonical reference](#).

Sum energy over operating phases

$$E = \sum P_i \Delta t = 1 \times 2 + 3 \times 4 + 0.5 \times 6 = 17 \text{ kWh}$$

1 — Concrete question

What quantity must be determined in “Exercise 8”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Total energy for a discrete profile is the sum of each phase's power multiplied by its duration

3 — Quantities first

E is total energy, $\sum$ means sum over operating phases, P_i is power during phase i , and Δt_i is the duration of that phase.

4 — Formula

$$E = \sum P_i \Delta t = 1 \times 2 + 3 \times 4 + 0.5 \times 6 = 17 \text{ kWh}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $E = \sum P_i \Delta t = 1 \times 2 + 3 \times 4 + 0.5 \times 6 = 17 \text{ kWh}$.

6 — Symbols

E is total energy, $\sum$ means sum over operating phases, P_i is power during phase i , and Δt_i is the duration of that phase.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

E is in J, Wh, or kWh; P_i in W or kW; Δt_i in s or h. Each $P_i \Delta t_i$ term is an energy and all terms must be expressed compatibly before summation.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

This operation is used because the relationship directly represents the dependency studied in “Exercise 8”.

11 — Assumptions

Phases must cover the period without double counting and use consistent units.

12 — Unit check

E is in J, Wh, or kWh; P_i in W or kW; Δt_i in s or h. Each $P_i \Delta t_i$ term is an energy and all terms must be expressed compatibly before summation. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$E = \sum P_i \Delta t_i = 1 \times 2 + 3 \times 4 + 0.5 \times 6 = 17 \text{ kWh}$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

A second calculation path or inverse operation should recover the original quantity.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This sum builds an energy budget from an operations schedule.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Sum three phases: 2 kW for 3 h, 5 kW for 2 h and 1 kW for 4 h.

Guided solution — open after trying

$$E = 2 \times 3 + 5 \times 2 + 1 \times 4 = 6 + 10 + 4 = 20 \text{ kWh.}$$

Independent exercise. Calculate 1.5 kW for 6 h then 4 kW for 1.5 h.

Independent solution — open after trying

$$E = 1.5 \times 6 + 4 \times 1.5 = 9 + 6 = 15 \text{ kWh.}$$

21 — Mission decision

This sum builds an energy budget from an operations schedule.

Physical reading

An integral accumulates a quantity over time or space. In operations it turns power into energy, flow into mass, or concentration into cumulative exposure under a chosen model.

A load of 8 kW for 2 h and then 3 kW for 6 h consumes $16 + 18 = 34$ kWh. Graphically this is the area under $P(t)$.
[Canonical reference.](#)

Power integral and discrete approximation

$$E = \int P \, dt \approx \sum P_i \Delta t_i$$

1 — Concrete question

What quantity must be determined in “Physical reading”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

When power varies continuously, energy is the accumulated area under the power curve; a sum over short intervals approximates it

3 — Quantities first

E is accumulated energy, $\int$ is continuous integration, P is instantaneous power, t is time, Σ is the discrete sum, P_i is power on interval i , and Δt_i is that interval duration.

4 — Formula

$$E = \int P \, dt \approx \Sigma P_i \Delta t_i$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $E = \int P \, dt \approx \Sigma P_i \Delta t_i$.

6 — Symbols

E is accumulated energy, $\int$ is continuous integration, P is instantaneous power, t is time, Σ is the discrete sum, P_i is power on interval i , and Δt_i is that interval duration.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

E is in J, Wh, or kWh; P and P_i in W or kW; t and Δt_i in s or h within a coherent unit system. Power×time has the dimension of energy.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “Physical reading”.

11 — Assumptions

The discrete approximation needs sufficiently fine steps when power changes rapidly.

12 — Unit check

E is in J, Wh, or kWh; P and P_i in W or kW; t and Δt_i in s or h within a coherent unit system. Power×time has the dimension of energy. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

By discrete approximation: $6 \text{ kW} \times 2 \text{ h} + 4 \text{ kW} \times 3 \text{ h} + 2 \text{ kW} \times 5 \text{ h} = 34 \text{ kWh}$.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This relation links power telemetry to actual energy consumed.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Approximate the power integral with three steps: 6 kW for 2 h, 4 kW for 3 h, 2 kW for 5 h.

Guided solution — open after trying

$$E \approx \Sigma P_i \Delta t_i = 6 \times 2 + 4 \times 3 + 2 \times 5 = 12 + 12 + 10 = 34 \text{ kWh}.$$

Independent exercise. With 7 kW for 1 h then 3 kW for 4 h, calculate energy.

Independent solution — open after trying

$$E \approx 7 \times 1 + 3 \times 4 = 7 + 12 = 19 \text{ kWh.}$$

21 — Mission decision

This relation links power telemetry to actual energy consumed.

Adding power values without their durations has no energy unit and sizes no battery.

9. Exponentials and logarithms: think in ratios

Exponential behavior appears when a rate depends on the amount already present. A logarithm is the inverse operation. The practical lesson is that exponential behavior can cross orders of magnitude very quickly.

Logarithmic axes are useful when values span large ratios. A straight line on log axes carries different meaning from a straight line on linear axes.

Move into calculation

Exponentials describe changes proportional to the current state; logarithms invert those relationships or compare orders of magnitude. They appear in decay, kinetics, reliability, and some thermal or biological models.

With an ideal 2% degradation per cycle, the remaining fraction after 20 cycles is $0.98^{20} \approx 0.668$. It is not a 40% remainder from simple subtraction because the base changes each cycle. The teaching value is seeing units turn into a decision.

[Canonical reference.](#)

Accumulation of a repeated fractional loss

$$f_n = (1-r)^n = 0.98^{20} \approx 0.668$$

1 — Concrete question

What quantity must be determined in “Move into calculation”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A small loss repeated many times compounds multiplicatively and can become significant

3 — Quantities first

f_n is the fraction remaining after n stages, r is the fractional loss at each stage between 0 and 1, and n is the dimensionless number of stages or cycles.

4 — Formula

$$f_n = (1-r)^n = 0.98^{20} \approx 0.668$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $f_n = (1-r)^n = 0.98^{20} \approx 0.668$.

6 — Symbols

f_n is the fraction remaining after n stages, r is the fractional loss at each stage between 0 and 1, and n is the dimensionless number of stages or cycles.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

f_n and r are dimensionless fractions; n is a dimensionless count.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

Here the operation expresses the following mechanism: the relationship directly represents the dependency studied in “Move into calculation”.

11 — Assumptions

The same rate is assumed for each cycle and cycles are treated as successive.

12 — Unit check

f_n and r are dimensionless fractions; n is a dimensionless count. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

$$f_n = (1-r)^n = 0.98^{20} \approx 0.668$$

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An inverse calculation should reconstruct one input within rounding error.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This law shows why efficiency slightly below one hundred percent can strongly affect a long campaign.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. A capacity loses 3% each cycle for 10 cycles. Calculate the remaining fraction.

Guided solution — open after trying

$$f_n = (1-0.03)^{10} = 0.97^{10} \approx 0.737. \text{ About 73.7\% remains, so cumulative loss is } \approx 26.3\%.$$

Independent exercise. With 1% loss over 50 cycles, calculate the remaining fraction.

Independent solution — open after trying

$$0.99^{50} \approx 0.605. \text{ About 60.5\% remains.}$$

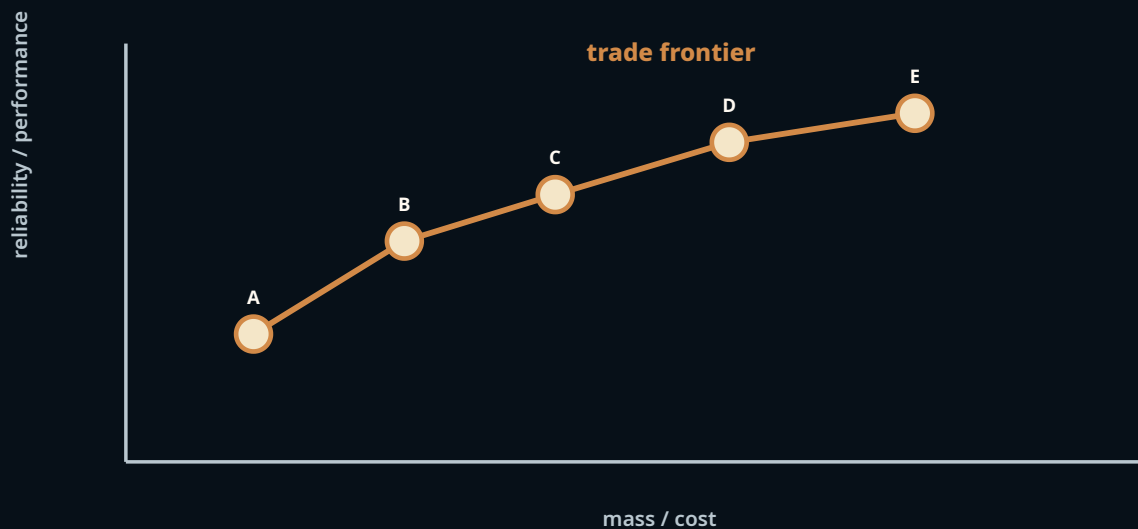
21 — Mission decision

This law shows why efficiency slightly below one hundred percent can strongly affect a long campaign.

An exponential model must be justified by the mechanism; visual resemblance alone is not enough.

Space Academy · 01

Optimisation multi-objectif / multi-objective optimization



A constraint eliminates; a preference trades

Optimization trade: expose competing criteria before choosing an architecture.

10. Uncertainty: worst-case and independent errors

Ten interfaces at ± 0.1 mm can stack to ± 1.0 mm in arithmetic worst case. If errors are genuinely independent and centered, root-sum-square gives $\sqrt{10} \times 0.1 \approx 0.316$ mm. [Canonical reference](#).

The results reflect different assumptions. A common calibration bias does not disappear through square-root statistics. State the error model with the number.

Case study — uncertainty belongs to the decision

An assembly contains four measured dimensions with independent uncertainties of ± 0.20 , ± 0.10 , ± 0.15 , and ± 0.05 mm. Worst-case arithmetic adds amplitudes to ± 0.50 mm. If errors are genuinely independent, centred, and suitable for root-sum-square combination, the RSS value is about 0.274 mm. These numbers belong to different models; choosing the smaller one simply because it is convenient is not valid reasoning.

In a Mars workshop, a shared calibration bias can destroy the independence assumption. Four instruments calibrated from the same biased reference can all shift in the same direction. Uncertainty therefore belongs to measurement architecture: independent references, cross-checks, and calibration intervals. The equation never erases the physical structure of error sources.

Worst-case sum of uncertainties

worst case = $0.20 + 0.10 + 0.15 + 0.05 = 0.50$ mm; RSS ≈ 0.274 mm

1 — Concrete question

What quantity must be determined in “10. Uncertainty: worst-case and independent errors”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

If all possible errors push in the same direction, their sum gives a conservative bound

3 — Quantities first

Worst case is the direct sum of the absolute uncertainty contributions. RSS is the root-sum-square result used when the contributions are treated as independent.

4 — Formula

Worst case = $0.20+0.10+0.15+0.05 = 0.50$ mm; RSS ≈ 0.274 mm

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: worst case = $0.20+0.10+0.15+0.05 = 0.50$ mm; RSS ≈ 0.274 mm.

6 — Symbols

Worst case is the direct sum of the absolute uncertainty contributions. RSS is the root-sum-square result used when the contributions are treated as independent.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

All uncertainty contributions must use the same unit, here mm; both the worst-case sum and RSS therefore return mm.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “10. Uncertainty: worst-case and independent errors”.

11 — Assumptions

This bound assumes a deliberately adverse combination and does not describe probability.

12 — Unit check

All uncertainty contributions must use the same unit, here mm; both the worst-case sum and RSS therefore return mm. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

Worst case = $0.20+0.10+0.15+0.05 = 0.50$ mm; RSS ≈ 0.274 mm

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

Worst case provides a simple safety bound when consequences of exceedance are severe.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Add worst-case contributions ± 0.12 , ± 0.08 and ± 0.05 mm.

Guided solution — open after trying

$u_{\text{worst}} = 0.12+0.08+0.05 = 0.25$ mm. This assumes all errors can align adversely.

Independent exercise. Calculate worst case for ± 0.20 , ± 0.15 , ± 0.10 and ± 0.05 mm.

Independent solution — open after trying

$0.20+0.15+0.10+0.05 = 0.50$ mm.

21 — Mission decision

Worst case provides a simple safety bound when consequences of exceedance are severe.

Control point

Uncertainties can be combined as a worst case or, under independence assumptions, by root-sum-square. The choice depends on the safety question and the nature of the errors.

Ten interfaces at ± 0.1 mm give ± 1.0 mm by arithmetic worst-case stacking. If errors are independent, centered, and comparable, root-sum-square gives $\sqrt{10 \times 0.1^2} \approx 0.316$ mm. An engineer keeps numerical result, margin, and validity domain separate. [Canonical reference](#).

Root-sum-square combination of independent uncertainties

$$u_{\text{RSS}} = \sqrt{\sum u_i^2}$$

1 — Concrete question

What quantity must be determined in “Control point”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

Independent uncertainties do not always add linearly; a root-sum-square combination better represents their collective spread

3 — Quantities first

u_{RSS} is the root-sum-square combined uncertainty, u_i is uncertainty contribution i expressed in the same unit as the final quantity, and Σ means sum over all contributions.

4 — Formula

$$u_{\text{RSS}} = \sqrt{\sum u_i^2}$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $u_{\text{RSS}} = \sqrt{\sum u_i^2}$.

6 — Symbols

u_{RSS} is the root-sum-square combined uncertainty, u_i is uncertainty contribution i expressed in the same unit as the final quantity, and Σ means sum over all contributions.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

All u_i use the same physical unit. u_i^2 has unit^2 , the sum retains unit^2 , and the square root returns u_{RSS} to the original unit. Relative uncertainties are dimensionless.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “Control point”.

11 — Assumptions

Independence and the statistical meaning of each term must be justified.

12 — Unit check

All u_i use the same physical unit. u_i^2 has unit^2 , the sum retains unit^2 , and the square root returns u_{RSS} to the original unit. Relative uncertainties are dimensionless. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

For independent uncertainties 0.20, 0.15 and 0.10 mm, $u_{\text{RSS}} = \sqrt{(0.20^2 + 0.15^2 + 0.10^2)} \approx 0.269$ mm.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

This combination avoids excessive conservatism while preserving quantitative traceability.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Combine 0.20, 0.15 and 0.10 mm by RSS.

Guided solution — open after trying

$u_{\text{RSS}} = \sqrt{(0.04 + 0.0225 + 0.01)} = \sqrt{0.0725} \approx 0.269$ mm. This is below the arithmetic sum 0.45 mm.

Independent exercise. Combine four independent 0.10 mm uncertainties.

Independent solution — open after trying

$u_{\text{RSS}} = \sqrt{(4 \times 0.10^2)} = \sqrt{0.04} = 0.20$ mm.

21 — Mission decision

This combination avoids excessive conservatism while preserving quantitative traceability.

A common machining bias is not independent; using RSS on correlated errors underestimates possible drift.

11. Optimization: objectives conflict

Minimizing mass, power, time, cost, and risk while maximizing margin and repairability is a multiobjective problem. Often no single design dominates all alternatives.

Separate hard constraints from weighted preferences and test sensitivity. If the winner changes for a tiny weight change, the decision is fragile.

Interpretation

Optimization means choosing among conflicting objectives. Mass, power, volume, cost, reliability, crew time, and repairability usually do not share one common minimum.

Adding a backup unit increases installed mass and power yet can reduce downtime. A simple objective function might weight mass, energy, and risk, but those weights are themselves a decision. The formula is useful because it makes a system dependency visible.

Weighted multi-criteria score

$$J = w_m M + w_E E + w_R R$$

1 — Concrete question

What quantity must be determined in “Interpretation”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A weighted score combines several criteria while explicitly giving more or less importance to each

3 — Quantities first

J is the objective score; w_m , w_E , and w_R are weighting coefficients; M is the mass metric; E is the energy metric; R is the third explicitly defined performance metric for this trade study.

4 — Formula

$$J = w_m M + w_E E + w_R R$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $J = w_m M + w_E E + w_R R$.

6 — Symbols

J is the objective score; w_m , w_E , and w_R are weighting coefficients; M is the mass metric; E is the energy metric; R is the third explicitly defined performance metric for this trade study.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

J uses the chosen score convention. The weights are dimensionless only if M, E, and R have already been normalized; otherwise the coefficients must carry the units needed to make the three summed terms compatible.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

The calculation is structured this way because the relationship directly represents the dependency studied in “Interpretation”.

11 — Assumptions

Criteria must be normalized compatibly and weights must be justified rather than hidden.

12 — Unit check

J uses the chosen score convention. The weights are dimensionless only if M, E, and R have already been normalized; otherwise the coefficients must carry the units needed to make the three summed terms compatible. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

With $w = (0.40, 0.35, 0.25)$ and scores (8, 6, 9), $J = 0.40 \times 8 + 0.35 \times 6 + 0.25 \times 9 = 7.55$.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

An independent check substitutes the result back into the starting relationship.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

The score structures a comparison, but the final decision must remain readable criterion by criterion.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Calculate a weighted score with weights 0.40/0.35/0.25 and scores 8/6/9.

Guided solution — open after trying

$$J = 0.40 \times 8 + 0.35 \times 6 + 0.25 \times 9 = 3.20 + 2.10 + 2.25 = 7.55. \text{ Weights sum to 1.}$$

Independent exercise. With weights 0.50/0.30/0.20 and scores 7/9/5, calculate J.

Independent solution — open after trying

$$J = 0.50 \times 7 + 0.30 \times 9 + 0.20 \times 5 = 3.5 + 2.7 + 1.0 = 7.2.$$

The score structures a comparison, but the final decision must remain readable criterion by criterion.

A precise numerical minimum is not a physical truth when weights, constraints, or scenarios were chosen arbitrarily.

Sensitivity check for 11. Optimization: objectives conflict. Change one input at a time and sketch the new point on the same graph. Decide whether the local slope stays constant; if it does not, state the interval over which your linear approximation remains useful. Finish by naming the variable whose uncertainty most strongly changes the engineering decision.

12. Mini-project: water buffer with margin

Assume 40 kg/day makeup, 30 days autonomy, and 25% planning margin. Nominal inventory = 1,200 kg; with margin = 1,500 kg. At roughly 1,000 kg/m³, that is 1.5 m³ water. [Canonical reference](#).

Tank design still needs ullage, structure, insulation, quality control, plumbing, leaks, and refill strategy. Mathematics frames the requirement; engineering completes the system.

Guided project — size a buffer without hiding assumptions

A base wants a cylindrical water buffer able to cover a production interruption. Assume 100 people, a net loss of 0.45 kg/person/day after recycling, and a target endurance of 12 days. Useful requirement is $100 \times 0.45 \times 12 = 540$ kg, about 0.54 m³ using the teaching density 1,000 kg/m³. Adding a 30% operational margin gives 702 kg or 0.702 m³. Keep that margin separate from uncertainty in the actual loss rate. [Canonical reference](#).

Choose a vertical cylindrical tank with 1.0 m internal diameter, radius 0.5 m. Base area is $\pi \times 0.5^2 \approx 0.785$ m². For 0.702 m³, useful liquid height is $V/A \approx 0.894$ m. If 15% free volume is reserved above the liquid, minimum geometric volume becomes $0.702/0.85 \approx 0.826$ m³, corresponding to total height about 1.052 m. Geometry has translated an inventory requirement into a physical dimension. [Canonical reference](#).

Now give the 0.45 kg/person/day net loss a simplified uncertainty of ± 0.08 . The high case, 0.53, requires 636 kg before margin. With 30%, inventory becomes 826.8 kg. The previous tank is no longer comfortable: 0.826 m³ is almost exactly the liquid volume required before free volume is reserved. Sizing is therefore sensitive to uncertainty in loss rate, not merely to precise cylinder dimensions. [Canonical reference](#).

The next mathematical question is not automatically 'increase diameter.' A larger diameter lowers height but changes floor area, some wall areas, structure, and access. A taller cylinder saves floor footprint while creating other constraints. Optimization requires objectives: mass, volume, accessibility, stability, fabrication, cleaning. Without an objective function, 'optimal' has no defined meaning.

Finish with a sensitivity curve by calculating inventory for 0.35, 0.45, 0.55, and 0.65 kg/person/day. The result is a straight line because the model is linear in loss rate. If the real system contains a threshold, saturation, or efficiency that varies with flow, the curve will no longer be linear. That is the purpose of mathematics here: turn verbal requirements into equations and expose which assumptions control the decision.

Water-buffer mini-project

$$M = N \times q \times t; V = M/\rho; h = V/(\pi r^2)$$

1 — Concrete question

What quantity must be determined in “12. Mini-project: water buffer with margin”, and how does this relationship calculate it without losing the physical meaning?

2 — Intuition without symbols

A cumulative human demand becomes water mass, then volume, then storage height in a chosen geometry

3 — Quantities first

M is total resource mass, N is the number of people, q is mass consumption per person per unit time, t is duration, V is the volume occupied by mass M, ρ is density, h is cylinder height, r is cylinder radius, and π is the dimensionless constant pi.

4 — Formula

$$M = N \times q \times t; V = M/\rho; h = V/(\pi r^2)$$

5 — Read aloud

Read the relationship from left to right, naming each quantity and operation: $M = N \times q \times t$; $V = M/\rho$; $h = V/(\pi r^2)$.

6 — Symbols

M is total resource mass, N is the number of people, q is mass consumption per person per unit time, t is duration, V is the volume occupied by mass M, ρ is density, h is cylinder height, r is cylinder radius, and π is the dimensionless constant pi.

7 — Pronunciation

Say the operations, subscripts and exponents explicitly so another person can reconstruct the relationship without seeing the page.

8 — Units

M is in kg; N is a person count; q in kg/(person·time); t in the matching time unit; V in m³; ρ in kg/m³; h and r in m; π is dimensionless.

9 — Convention

Keep the sign, time-base and system boundary defined in the section; do not change reference halfway through the calculation.

10 — Why this operation

This operation is used because the relationship directly represents the dependency studied in “12. Mini-project: water buffer with margin”.

11 — Assumptions

Consumption, duration, density and geometry must describe the same scenario, with margins handled separately.

12 — Unit check

M is in kg; N is a person count; q in kg/(person·time); t in the matching time unit; V in m³; ρ in kg/m³; h and r in m; π is dimensionless. The unit check must recover the physical dimension expected for the requested result.

13 — Numerical case

For $N = 60$, $q = 0.40$ kg/day, $t = 15$ days: $M = 360$ kg ≈ 0.36 m³ of water. With $r = 0.50$ m, $h = 0.36/(\pi \times 0.50^2) \approx 0.46$ m.

14 — Why each operation

Each operation corresponds to one reasoning step: it combines, compares or scales quantities exactly as required by the model.

15 — Algebra check

A second calculation path or inverse operation should recover the original quantity.

16 — Mental estimate

Round the inputs to one useful digit and check that the detailed result remains in the same order of magnitude.

17 — Interpretation

The chain turns an endurance requirement into physical storage dimensions.

18 — What it does not prove

This calculation proves neither universal model validity nor input-data quality; it answers only the declared scenario and assumptions.

19 — Sensitivity or limit case

Vary one input at a time to identify which assumption drives the result most strongly.

20 — Guided and autonomous practice

Guided exercise. Size a buffer for 60 people, 0.40 kg/person/day, 15 days, water at 1,000 kg/m³ and a tank radius 0.50 m.

Guided solution — open after trying

$$M = Nqt = 60 \times 0.40 \times 15 = 360 \text{ kg. } V = M/\rho = 0.36 \text{ m}^3. h = V/(\pi r^2) = 0.36/(\pi \times 0.25) \approx 0.458 \text{ m.}$$

Independent exercise. Repeat with $N=80$, $q=0.30$ kg/day, $t=10$ days and $r=0.40$ m.

Independent solution — open after trying

$$M=80 \times 0.30 \times 10 = 240 \text{ kg}; V=0.24 \text{ m}^3; h=0.24/(\pi \times 0.16) \approx 0.477 \text{ m}.$$

21 — Mission decision

The chain turns an endurance requirement into physical storage dimensions.

Resolve a vector into components

$$V_x = V \cos \theta; V_y = V \sin \theta$$

1 — Concrete question

How should “resolve a vector into components” be used without losing order of magnitude or the meaning of the calculated quantity?

2 — Intuition without symbols

An oriented quantity can be projected onto two perpendicular axes to show how much acts horizontally and how much acts vertically.

3 — Quantities first

V is vector magnitude, θ its angle from the chosen horizontal axis, and V_x , V_y the components.

4 — Formula

$$V_x = V \cos \theta; V_y = V \sin \theta$$

5 — Read aloud

Read the relationship by naming every operation: $V_x = V \cos \theta; V_y = V \sin \theta$.

6 — Symbols

V is vector magnitude, θ its angle from the chosen horizontal axis, and V_x , V_y the components.

7 — Pronunciation

Name subscripts, exponents, angles and functions explicitly before entering numbers.

8 — Units

All components carry the same unit as V ; trigonometric functions are dimensionless.

9 — Convention

Keep the sign, angle and significant-figure convention stated in the course.

10 — Why this operation

The relationship isolates the useful mathematical structure so scale or direction is handled without ambiguity.

11 — Assumptions

The angle must be measured from the stated axis with a consistent sign convention and genuinely perpendicular axes.

12 — Unit check

All components carry the same unit as V ; trigonometric functions are dimensionless. The left and right sides must describe the same physical dimension.

13 — Numerical case

For $V = 10 \text{ m/s}$ and $\theta = 30^\circ$, $V_x \approx 8.66 \text{ m/s}$ and $V_y = 5.00 \text{ m/s}$.

14 — Why each operation

Each operation corresponds to an explicit mathematical property; it must not be replaced by an unexplained decimal shift or calculator keystroke.

15 — Algebra check

Work backward from the calculated form to the starting form to check the algebra independently of numerical entry.

16 — Mental estimate

Before calculating, bound the answer with an order of magnitude or simple geometry to catch a gross error.

17 — Interpretation

Use components for slope, tilted thrust, line of sight or velocity before any vector addition.

18 — What it does not prove

The calculation validates neither the input data nor the physical model that led to this relationship.

19 — Sensitivity or limit case

Change one input at a time to see whether the result responds linearly, quadratically or through a change of direction.

20 — Guided and autonomous practice

Guided exercise. Resolve $V = 12 \text{ m/s}$ at $\theta = 60^\circ$.

Guided solution — open after trying

$V_x = 12 \cos 60^\circ = 6.0 \text{ m/s}$; $V_y = 12 \sin 60^\circ \approx 10.39 \text{ m/s}$. $\sqrt{6^2 + 10.39^2} \approx 12 \text{ m/s}$, checking the decomposition.

Independent exercise. Resolve $V = 20 \text{ m/s}$ at 45° .

Independent solution — open after trying

$V_x = V_y = 20/\sqrt{2} \approx 14.14 \text{ m/s}$.

21 — Mission decision

Use components for slope, tilted thrust, line of sight or velocity before any vector addition.

13. End-of-module check

- I isolate an unknown and check units.
- I know when proportional scaling is risky.
- I compute basic geometry and vector components.
- I read derivatives as rates and integrals as accumulation.
- I distinguish worst-case from statistical uncertainty.

Corrected drill set — independent check

1. Does cylinder volume merely double when radius doubles?

Not if height stays fixed: $V = \pi r^2 h$, so doubling radius multiplies volume by four. If radius and height both double, volume grows by eight. [Canonical mini-lesson](#).

2. A rover travels 3 km east then 4 km north. Displacement?

Displacement magnitude is $\sqrt{3^2 + 4^2} = 5 \text{ km}$ while path length is 7 km. They answer different physical questions. [Canonical mini-lesson](#).

3. A constant flow is 2 kg/h for 7.5 h. Accumulated mass?

For constant flow, the integral is rate $\times$ time: $2 \times 7.5 = 15 \text{ kg}$. If flow varies, sum intervals or integrate $q(t)$. [Canonical reference](#).

4. Does $\pm 0.2 \text{ mm}$ repeated five times always mean $\pm 1 \text{ mm}$?

$\pm 1 \text{ mm}$ is arithmetic worst case if every error aligns. Independent centred errors may support a smaller RSS model, but independence must be justified rather than assumed.

5. What does “optimize” mean without a criterion?

Nothing unique. Minimizing mass, energy, maintenance burden, or risk can produce different solutions. Define variables, constraints, and objective before claiming an optimum.

Deep practice workshop

For these problems, sketch the mathematical relationship before substituting numbers. State what is assumed linear, what scales with an area or volume, and what is a constraint rather than an unknown. The solutions focus on choosing the model and testing sensitivity, not merely on producing a final number.

1. Isolate a variable

From $E = P t$, isolate P and calculate average power for 72 kWh delivered in 9 h. [Canonical mini-lesson](#).

Reasoned solution: $P = E/t = 72/9 = 8 \text{ kW}$. Here kWh/h reduces to kW. [Canonical reference](#).

2. Cylinder

Tank $r=1.5$ m, $L=4$ m. Volume? [Canonical reference](#).

Reasoned solution: $V=\pi r^2 L \approx \pi \times 2.25 \times 4 \approx 28.27$ m³. Usable liquid capacity may be smaller if gas ullage is required. [Canonical mini-lesson](#). [Canonical reference](#).

3. Slope

What elevation change corresponds to 250 m horizontal at 6°?

Reasoned solution: $\Delta z = 250 \tan 6^\circ \approx 26.3$ m. If 250 m were distance along the slope, $\sin 6^\circ$ would be used instead. [Canonical reference](#).

Tangent applies because the stated 250 m is horizontal distance. If the problem had given distance along the slope, sine would be the relevant relation: identifying the triangle is part of the calculation.

4. Vectors

5 km east then 12 km north: displacement and path length?

Reasoned solution: Path length=17 km; displacement= $\sqrt{(25+144)}=13$ km. They answer different questions. [Canonical reference](#).

5. Integration

A load is 4 kW for 3 h and 1 kW for 9 h.

Reasoned solution: $E=12+9=21$ kWh. Energy is the area under the power-time curve. [Canonical reference](#).

6. Tolerance stack

Six interfaces ± 0.2 mm: worst case and independent RSS?

Reasoned solution: Worst case ± 1.2 mm; RSS= $0.2\sqrt{6} \approx 0.490$ mm. RSS assumes centered independent errors. [Canonical reference](#).

Worst-case addition assumes every tolerance drifts in the same adverse direction; RSS assumes independent, centered errors. Choosing between them is an engineering assumption, not a cosmetic mathematical preference.

7. Exponential

Capacity retains 99% each cycle. Fraction after 50 cycles?

Reasoned solution: $0.99^{50} \approx 0.605$. Repeated 1% loss is not subtraction of fifty percentage points. [Canonical reference](#).

Repeated loss compounds multiplicatively. The remaining fraction after fifty cycles shows why a small per-cycle degradation can dominate long-duration performance and why the cycles must actually be comparable.

8. Optimization capstone

Two architectures: A=1,000 kg, 20 kW, 4% unavailability; B=1,200 kg, 16 kW, 1% unavailability. Why is there no “best” answer without a criterion? [Canonical reference](#).

Reasoned solution: A minimizes mass while B reduces power and unavailability. Constraints and weights must be stated—for example maximum mass, available power, and risk penalty. Optimization starts by defining the decision, not by launching a solver.

No architecture is best without constraints. Mass, power, and unavailability have different units, so a decision needs hard limits or an explicitly justified weighting method rather than a single unexplained score.

Additional advanced problems

1. Area-volume

If every dimension of a similar module doubles, how do area and volume change?

Reasoned solution: Area $\times 4$ and volume $\times 8$. Surface-to-volume ratio halves, which can change thermal loss and wall mass per habitable volume.

2. Discrete derivative

Inventory is 500, 480, 455 kg at $t=0,2,4$ h. Average rates? [Canonical reference](#).

Reasoned solution: -10 kg/h then -12.5 kg/h. Loss is accelerating; a single overall -11.25 kg/h average hides the trend.

3. Flow integral

Flow is 2 kg/h for 5 h then 0.5 kg/h for 10 h.

Reasoned solution: $M=10+5=15$ kg. Canonical reference.

4. Vector mini-project

A rover goes 4 km east, 3 km north, 4 km west. Net displacement?

Reasoned solution: Final displacement is 3 km north while 11 km was traveled. Energy depends on path and profile, not net displacement.

The rover returns to its original east-west coordinate but not to its starting point: displacement is 3 km north. Travel energy depends on the full 11 km path and its terrain, not on displacement alone.

Mathematical reasoning laboratory — choose the model before solving it

This R54 parity extension deepens the operational reasoning of the English curriculum without replacing the existing lesson. It is designed to make the learner explain assumptions, verify a calculation or chain, and convert the result into a decision.

Translate the physical question into variables

Mathematics becomes useful only after the physical question has been stated. Define what is known, what is unknown and which quantities are allowed to vary. A proportion is appropriate when one quantity scales linearly with another under the stated conditions; it is not a universal rule. A derivative describes a local rate of change, while an integral accumulates a rate over an interval. The learner should be able to explain in ordinary language why a chosen mathematical object matches the physical mechanism before manipulating symbols.

Distinguish linear, nonlinear and threshold behaviour

Many beginner errors come from extending a linear trend beyond its valid range. Tank volume may scale approximately with geometry, but pressure losses, heat transfer, reaction rates and failure probabilities can depend nonlinearly on state. Operational procedures also contain thresholds: below a certain reserve, an action may change discontinuously from normal operation to conservation mode. Plotting or tabulating a few points is often enough to reveal whether a straight-line assumption is reasonable. When a model is only local, state the interval in which it is being used.

Use derivatives as sensitivity, not only slope

A derivative can be read as sensitivity: how much the output changes when an input changes slightly. This interpretation is useful for margins. If a one-degree temperature change produces a large change in a process output, temperature control deserves attention. If the sensitivity is small across the operating range, another uncertainty may dominate. In numerical work, a finite difference can approximate this reasoning even when an analytic derivative is unavailable, provided the step size is chosen carefully and the approximation is checked for stability.

Use integrals to close inventories

An inventory changes because flows enter and leave. Integrating a flow over time converts a rate into an accumulated quantity. This is the mathematical backbone of water, oxygen, energy, thermal and logistics budgets. If the rate is not constant, multiplying one instantaneous value by the whole duration is unjustified. Break the interval into segments or use a numerical integration method that preserves the operational events that matter, such as a high-power EVA recharge period or a temporary loss of water recovery.

Treat uncertainty as a decision variable

Uncertainty should be propagated far enough to know whether it affects the decision. Worst-case addition is conservative when several bounded errors could align; root-sum-square reasoning may be appropriate for independent random contributions, but independence must be defended rather than assumed. Sensitivity analysis is often more informative than

a single uncertainty number because it identifies which measurement or design improvement would most increase confidence.

Optimize only after constraints are explicit

Optimization is not the search for the biggest or smallest number in isolation. A Mars architecture usually has several competing objectives: mass, power, reliability, crew time, maintainability and scientific value. Constraints define the feasible region. A mathematically optimal point that violates a safety reserve or depends on unavailable maintenance skill is not an engineering solution. The practical sequence is therefore constraints first, objective second, sensitivity third and verification last.

Progressive mastery drills — eight linked checks

▼ Drill 1 — Proportion

Identify when a direct proportion is justified and give one example where extrapolating it would fail.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 2 — Algebraic rearrangement

Rearrange a simple resource equation to solve for the required storage instead of endurance, preserving units.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 3 — Functions and graphs

Sketch or describe a response that increases linearly at first and then saturates; explain why one straight line cannot model the whole range.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 4 — Trigonometry

Use a right-triangle decomposition to explain how a line-of-sight or slope problem becomes horizontal and vertical components.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 5 — Vectors

Explain why adding two velocity magnitudes is not enough when their directions differ.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 6 — Derivative as sensitivity

Estimate a local sensitivity from two nearby operating points and state what a large slope means operationally.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 7 — Integral as accumulation

Explain how a variable power profile becomes total energy over a shift.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Drill 8 — Constrained optimization

Choose between two designs when one minimizes mass and the other preserves more power and maintenance margin; state the constraints first.

Expected reasoning. Show the physical meaning, units or evidence path, state at least one assumption, and explain what operational decision would change if the result or evidence were different.

▼ Integrated exercise — Build a one-variable sensitivity study

A simplified water buffer model starts with 500 L and loses 18 L/day net after recovery. Treat the net loss as uncertain between 15 and 22 L/day. Calculate the nominal endurance at 18 L/day and the two endpoint endurances. Then identify which extra measurement would most directly reduce uncertainty in the endurance estimate.

Reasoned solution. Nominal endurance is $500/18 \approx 27.8$ days. At 15 L/day it is about 33.3 days; at 22 L/day it is about 22.7 days. The spread is operationally large, so the useful next measurement is not another calculation: it is a better estimate of the net daily loss under the actual operating mode, including recovery performance and unavoidable discharge. [Canonical reference](#).

Primary-source bridge for this extension. [Official source 1](#) [Official source 2](#). These links are evidence anchors already present in the French reference module; values and limits must still be checked against the source version applicable to the mission.

Sources and references

These references stabilize units and measurement language; the geometries and numerical cases are deliberately constructed exercises used to teach the method.

- [NIST — Definitions of SI Base Units](#) — SI base units and exact defining constants.
- [NIST Technical Note 1297 — Measurement Uncertainty](#) — Framework for expressing measurement uncertainty.

First Man mathematical closure lab — model first, algebra second

Useful mathematics in mission work is not a collection of tricks. It is a disciplined way to translate a physical question into variables, choose a model whose assumptions fit the situation, solve it without losing units, and then test whether the answer behaves sensibly. This dossier strengthens the bridge between arithmetic and later orbital, thermal, logistics and reliability work.

Translate the question before touching the calculator

Begin by naming the unknown, the quantities already known and the constraints. A problem that says ‘the rover must reach a site and return’ is not yet an equation. It becomes one only after the path length, reserve policy, energy-per-kilometre model and usable battery energy are defined. Mathematics starts with representation, not symbol manipulation.

A good variable table prevents one symbol from quietly changing meaning. If d is outward distance in one paragraph and total round-trip distance in another, even correct algebra can produce a wrong operational conclusion. Keep the definition, unit and sign convention beside each variable until the model is stable.

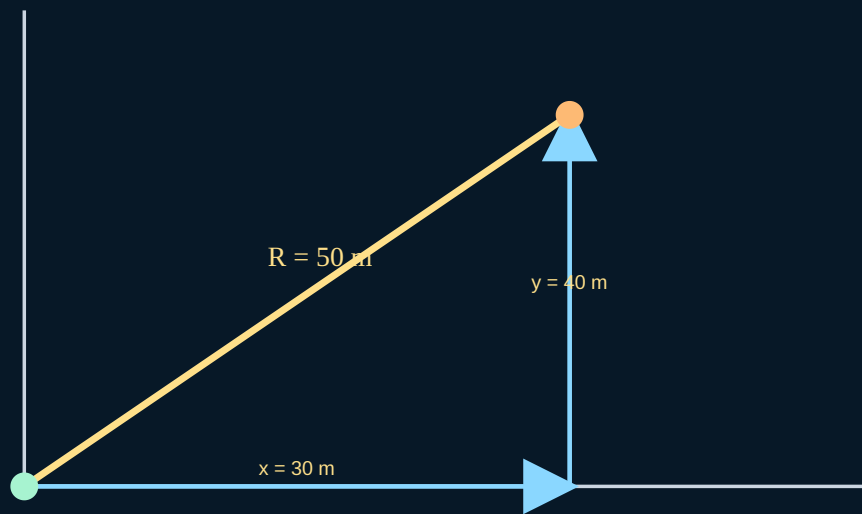
Choose linear, nonlinear or threshold behaviour deliberately

A rule of three is valid only when the relationship is proportional over the range of interest. Doubling crew size may double a consumable flow in a simple model, but it does not necessarily double radiator area, staffing depth or emergency-rescue complexity. Some variables scale with length, others with area or volume, and many systems have thresholds where a new pump, shift team or power bus becomes necessary.

Before extrapolating, ask what mechanism creates the relationship. If heat loss is dominated by surface area while habitable capacity follows volume, scaling a habitat changes their ratio. If a filter has a maximum flow, the behaviour may be nearly linear below that point and unacceptable above it. Model choice is therefore an engineering claim that needs justification.

Vector geometry map

Orthogonal components, resultant displacement and heading context.



Delta-Sierra pedagogical synthesis · schematic, not to scale

Orthogonal components, resultant displacement and heading context. Pedagogical synthesis by Delta-Sierra from the primary sources cited in this course; schematic, not to scale.

Vectors make direction part of the evidence

A velocity, force or displacement cannot always be represented by magnitude alone. Two 5 m/s velocity changes in different directions do not simply add to 10 m/s. Resolve vectors into components or use geometry so that direction is retained. This is the mathematical foundation for guidance, rendezvous, loads, wind effects and surface navigation.

Draw the coordinate axes before inserting numbers. State whether angles are measured clockwise or counter-clockwise, from north or east, from the local horizontal or another reference. A surprising number of navigation mistakes are coordinate-definition mistakes rather than arithmetic mistakes.

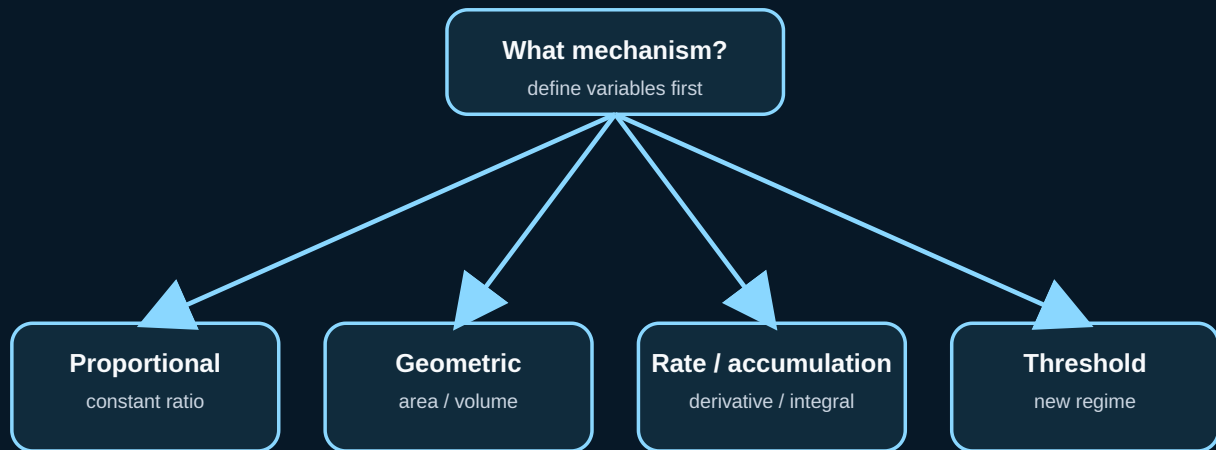
Derivatives and integrals answer different operational questions

A derivative asks how rapidly something changes at a moment or around a state. Battery state-of-charge decline, temperature rise and pressure change are rate questions. An integral accumulates a rate over time. Flow in litres per hour becomes inventory in litres only after accumulation over an interval. Keeping those meanings clear prevents the common mistake of treating a rate as if it were already a total.

For a beginner, numerical approximations are often enough. A slope between two measurements can estimate a rate; a table of power values multiplied by time intervals can estimate energy. The advanced mathematics comes later, but the conceptual distinction must be secure from the beginning.

Model-selection map

A decision tree for proportional, geometric, rate and threshold models.



Delta-Sierra pedagogical synthesis · schematic, not to scale

A decision tree for proportional, geometric, rate and threshold models. Pedagogical synthesis by Delta-Sierra from the primary sources cited in this course; schematic, not to scale.

Optimization begins after constraints are explicit

Optimization is not 'make one number as small as possible'. Mission designs trade mass, energy, time, risk, redundancy, maintainability and crew workload. Measurement and uncertainty practice from [NIST Technical Note 1297 — Measurement Uncertainty](#) reminds us that inputs themselves may be uncertain, so a mathematically sharp optimum can be operationally fragile.

A robust choice often sits away from a theoretical optimum because it preserves options. A route that saves 3% energy but uses nearly all rescue margin may be inferior to a slightly longer route. State the objective function, the hard constraints, the soft preferences and the uncertainty before comparing alternatives.

Worked modelling review — when a rule of three fails

A small surface greenhouse prototype uses 18 m² of floor area for four crop racks. A naive scale-up to twelve racks might multiply every subsystem by three. Floor area may scale roughly that way, but structural mass, aisle width, lighting distribution, cooling, nutrient plumbing and crew access may not. The mathematical review therefore begins by classifying each relationship. Rack count is discrete. Lighting power may be approximately proportional within one technology range. Exterior heat exchange depends on area. Internal volume depends on three-dimensional geometry. A pump can remain unchanged until a flow threshold is crossed, at which point the model jumps to a different configuration. [Canonical reference](#).

Students should write a small model register. For each equation, state whether it is proportional, geometric, thresholded, empirical or merely a conservative bound. This prevents algebra from acquiring authority it has not earned. If a rule is based on two measured operating points, do not extrapolate it tenfold without asking whether the same mechanism still dominates. If the model contains a square or square root, sketch how the output changes before trusting a calculator result.

Question	Candidate model	Review question
Lighting power vs rack count	Approximately linear	Does driver or thermal efficiency change?
Tank surface vs linear size	Area $\propto L^2$ Canonical reference .	Is shape preserved?

Question	Candidate model	Review question
Tank volume vs linear size	Volume $\propto L^3$ Canonical reference .	Are internal structures negligible?
Pump architecture vs flow	Threshold / piecewise	When is a second pump required?

Numerical thinking without hidden algebra

Take a storage tank that contains 1,200 L and loses 35 L/day after recovery. A beginner can compute endurance as $1,200 \div 35 \approx 34.3$ days. The more important mathematical training is to ask how the result changes when the loss rate is uncertain. At 30 L/day, endurance is 40 days; at 40 L/day, it is 30 days. The model is nonlinear in the loss rate because the rate sits in the denominator. Equal changes in loss do not create equal changes in endurance near every operating point. [Canonical reference](#).

Now add a threshold: operations require at least 300 L protected reserve. The usable stock for normal depletion is no longer 1,200 L but 900 L. Endurance to the decision threshold is $900 \div 35 \approx 25.7$ days. The arithmetic is simple; the modelling decision is not. The reserve policy changes the boundary of the problem. This is why variables and constraints must be defined before solving. [Canonical reference](#).

Review drills — move from explanation to operational judgement

1. **Model choice.** Give one Mars example each of proportional, area-scaling, volume-scaling and threshold behaviour.
2. **Graph reading.** Sketch a battery state-of-charge curve and identify where a slope, total change and threshold are read.
3. **Constraint audit.** Write a two-variable optimization problem where energy is minimized but rescue margin remains a hard constraint.
4. **Coordinate audit.** Define axes and signs for a local rover map so another operator could reproduce your vector calculation.

Final mathematical review — is the model more defensible than the algebra?

- Are the unknown and all inputs defined with units?
- Was proportionality justified rather than assumed?
- Are coordinate axes and signs explicit for vector work?
- Does a rate remain distinct from an accumulated quantity?
- Are hard constraints separated from optimization preferences?
- Was sensitivity tested before declaring one optimum?

A polished derivation can still answer the wrong physical question. The final check therefore begins with the model statement and ends with a sensitivity question: which assumption, coefficient or constraint would change the recommendation first? If that answer is unknown, the mathematics is not yet operationally mature.

Primary-source bridges used in this closure dossier

- [NIST — Definitions of SI Base Units](#)
- [NIST Technical Note 1297 — Measurement Uncertainty](#)

Closure rule. A mathematical solution is reviewable only when the model choice, variable definitions, assumptions and constraints are visible before the algebra.